UK FLUIDS NETWORK

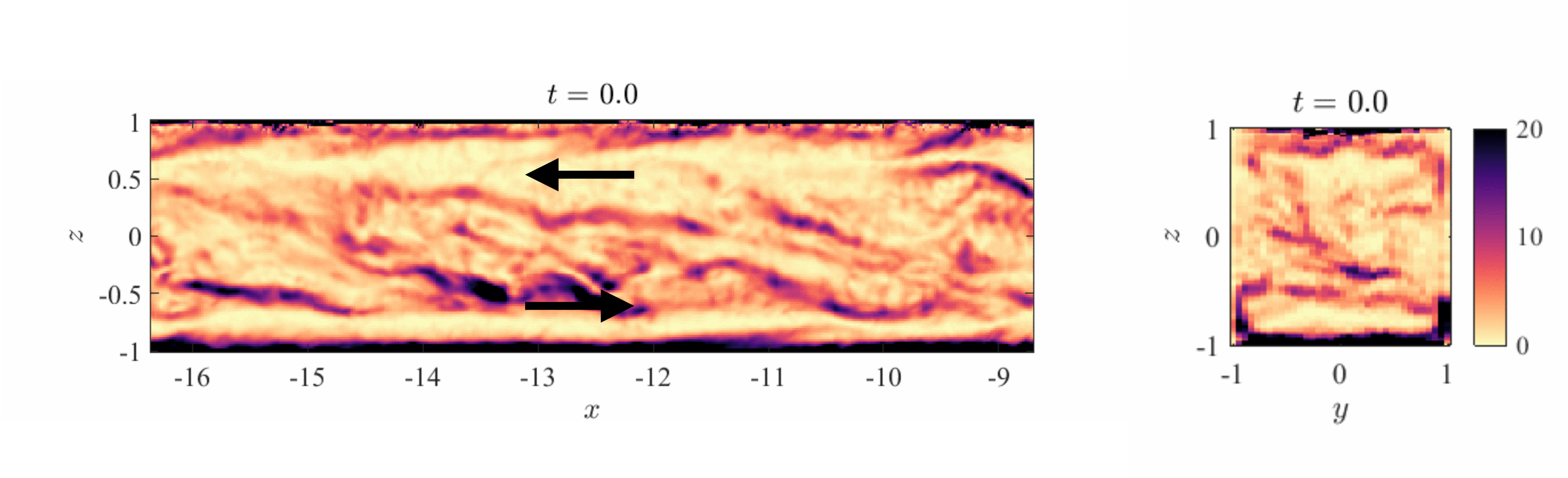
UK Fluids Conference, Cambridge
28 August 2019

Regime transitions and energetics of sustained stratified shear flows

Adrien Lefauve

Jamie Partridge

Paul Linden



MUST

Mathematical Underpinnings of Stratified Turbulence

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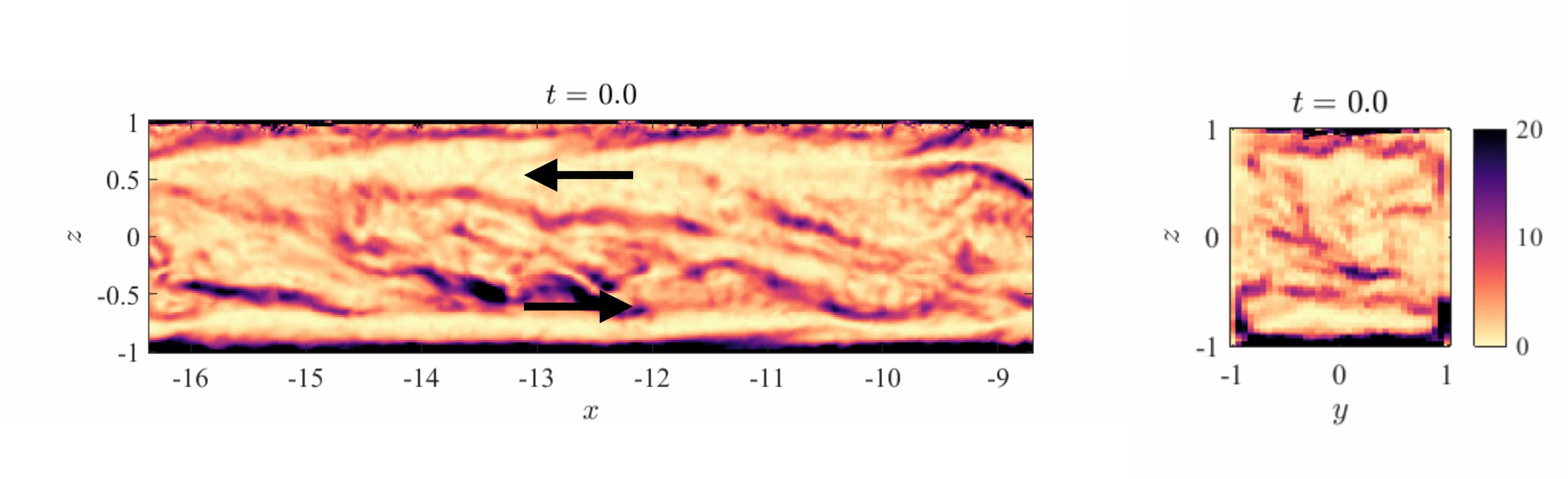

Mathematical Underpinnings of Stratified Turbulence

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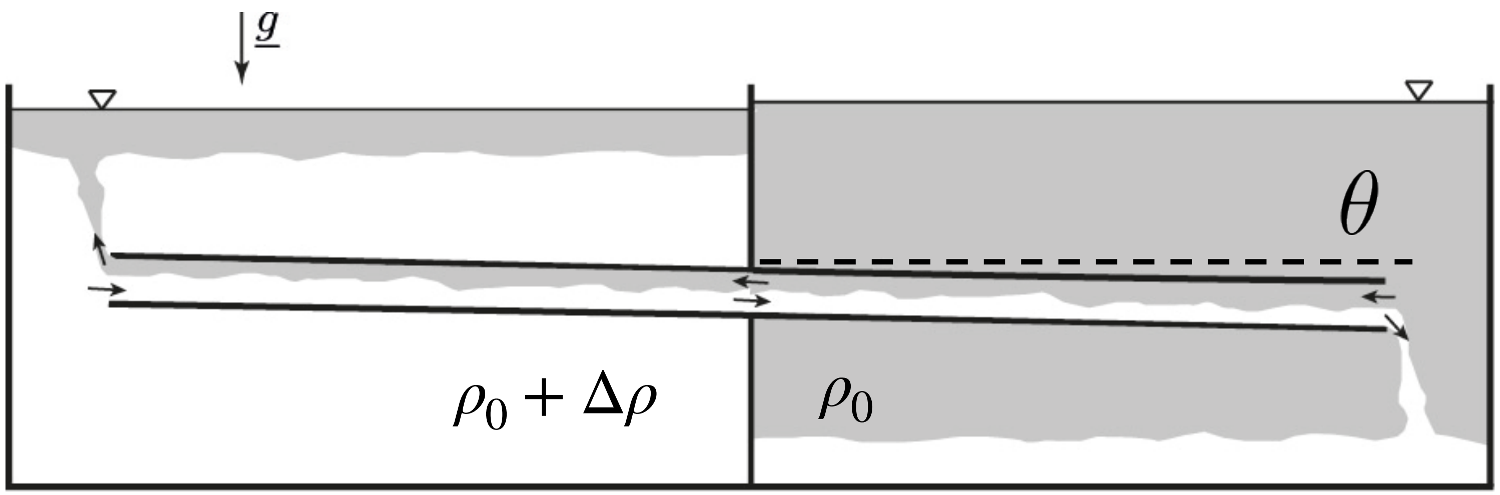
The Stratified Inclined Duct (SID) experiment

- Exchange flow between two reservoirs
- Two-layer **stratified shear flow** with **sustained forcing**



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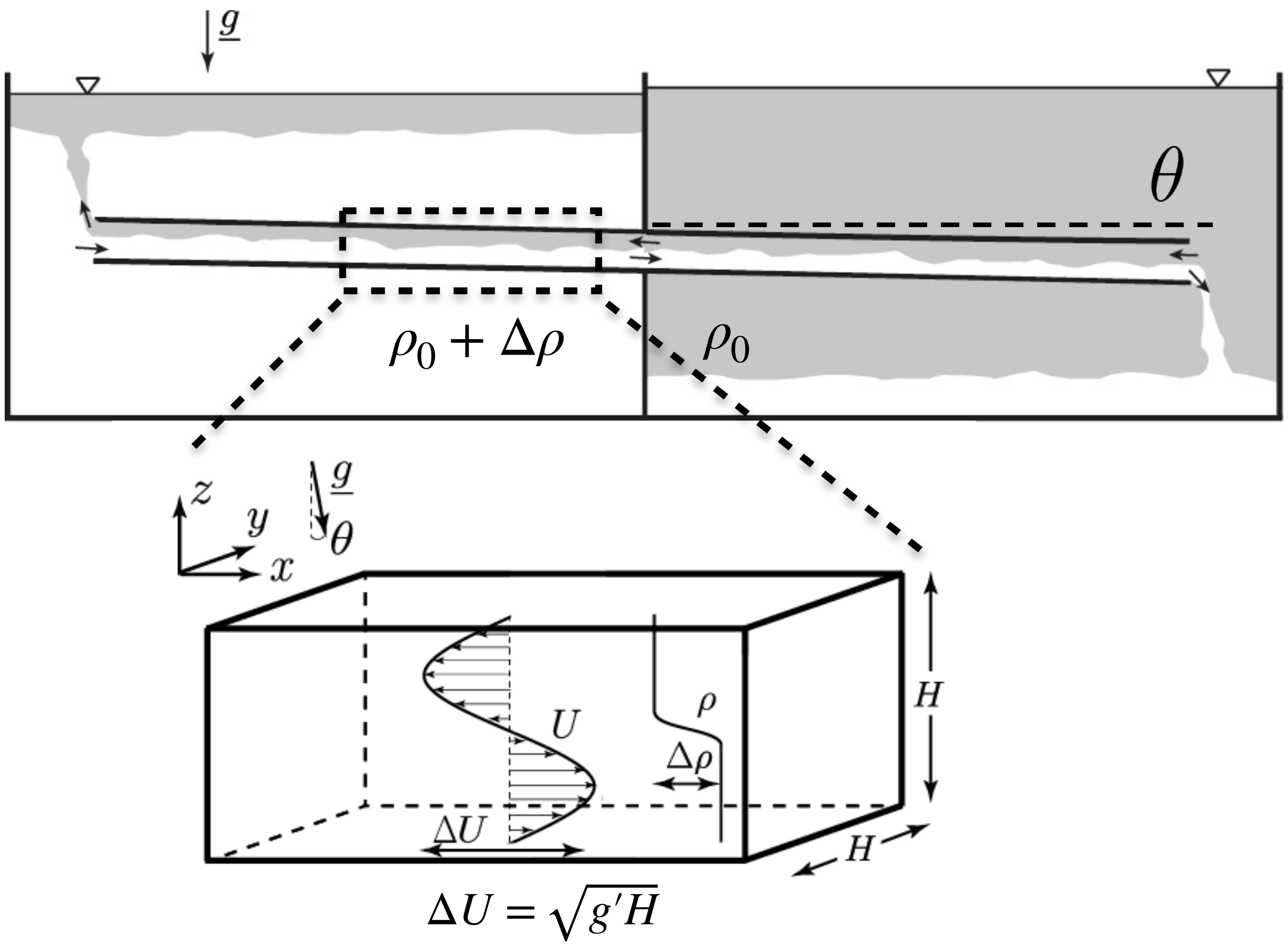

The Stratified Inclined Duct (SID) experiment

- Exchange flow between two reservoirs
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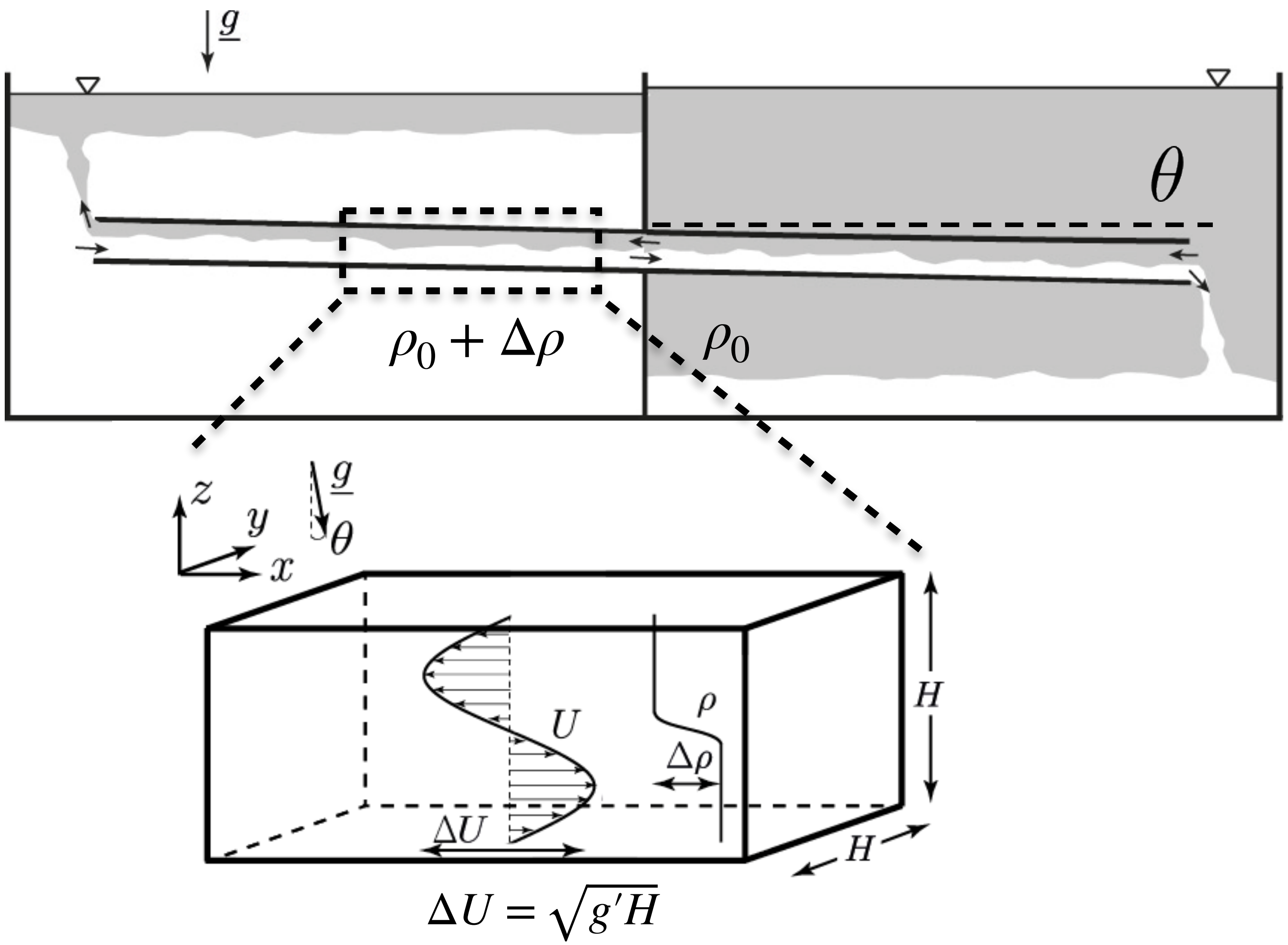
- Dimensional parameters

geometry	L	H
fluid	ν	κ
forcing	θ	$g' = g\dfrac{\Delta\rho}{\rho_0}$



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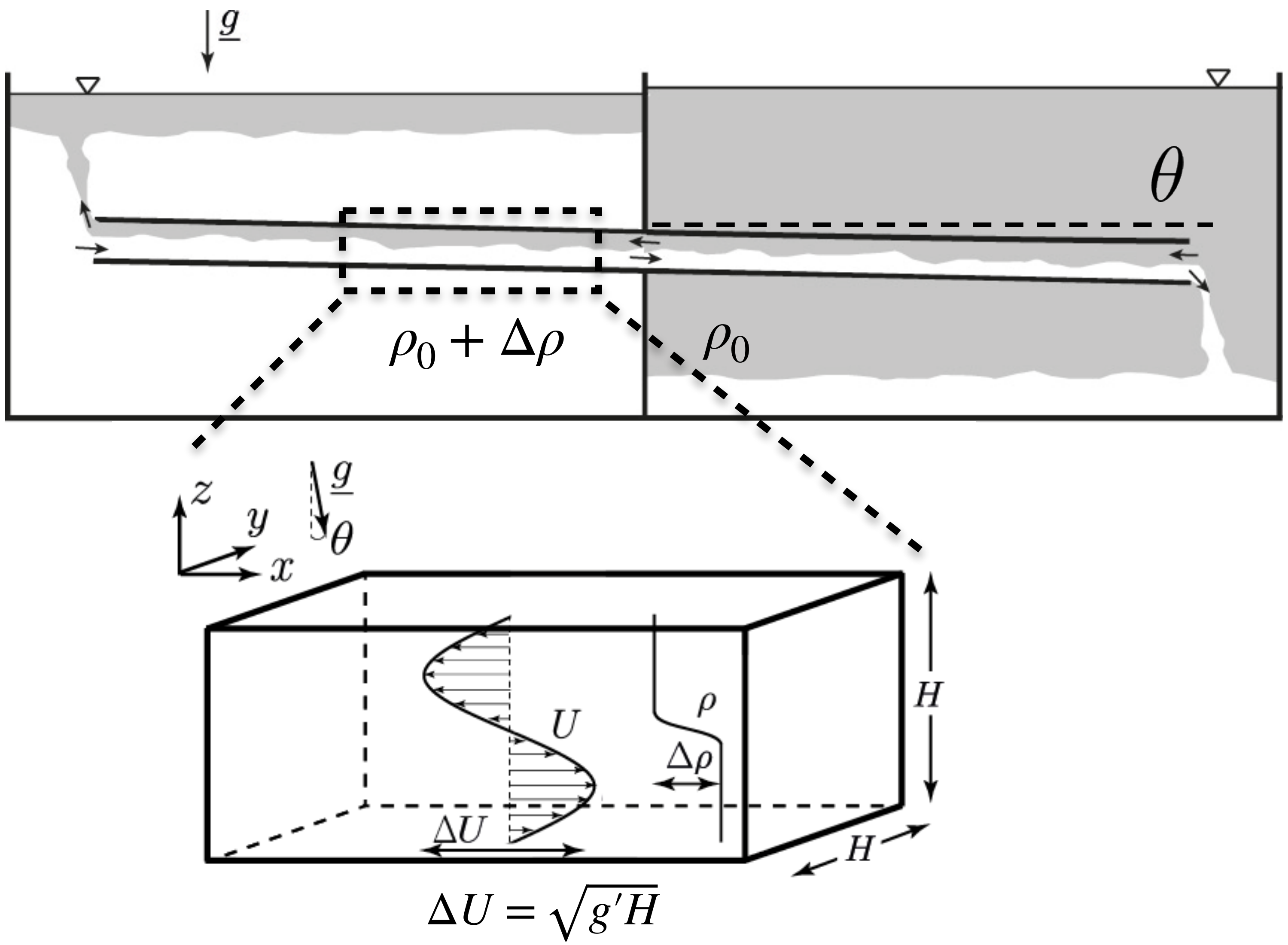

- Dimensional parameters

geometry	L	H
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- Non-dimensional parameters

Aspect ratio

$$A = \frac{L}{H} = 30$$

Prandtl number

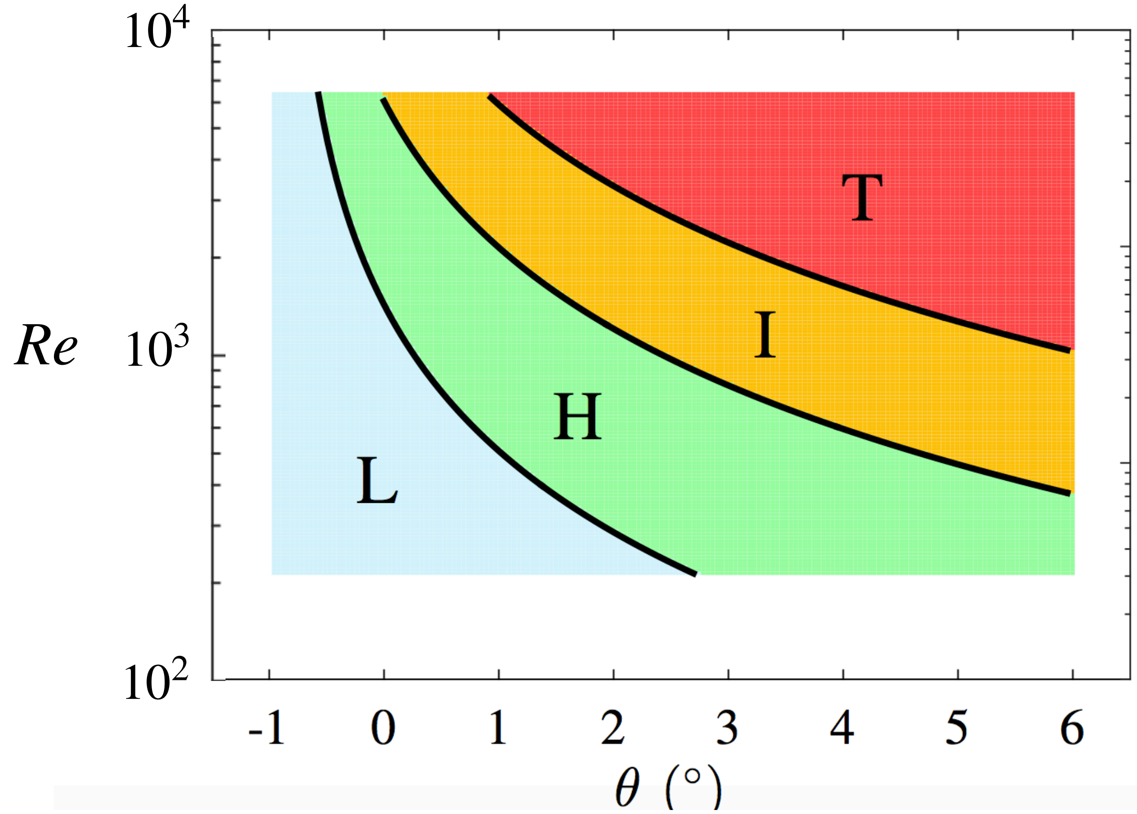
$$Pr = \frac{\nu}{\kappa} = 700$$

Forcing

$$\theta \qquad Re = \frac{\sqrt{g'H}H}{2\nu}$$

Flow regimes in the SID

Meyer & Linden (2014)



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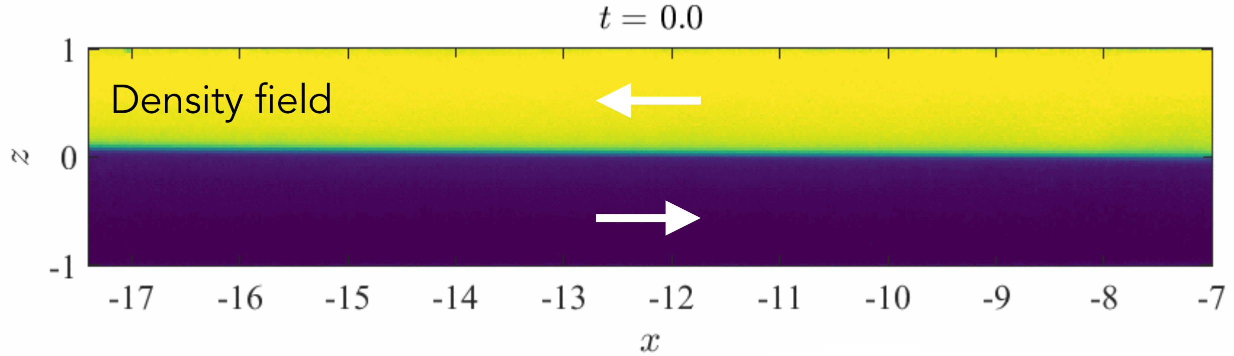

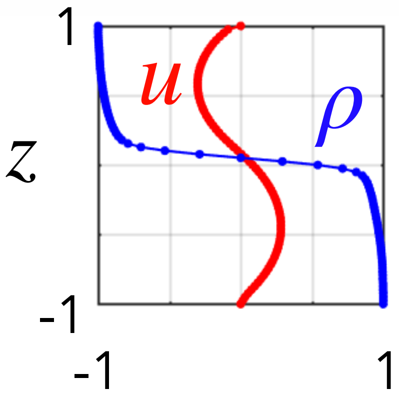

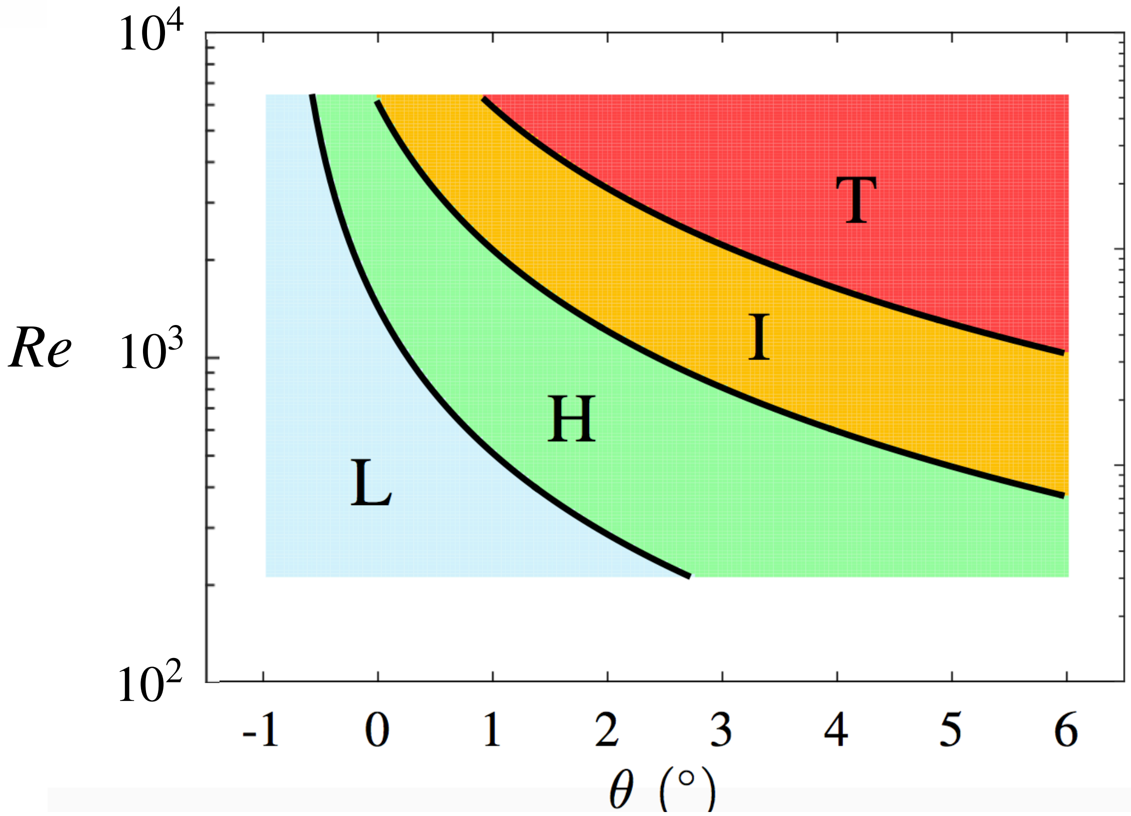

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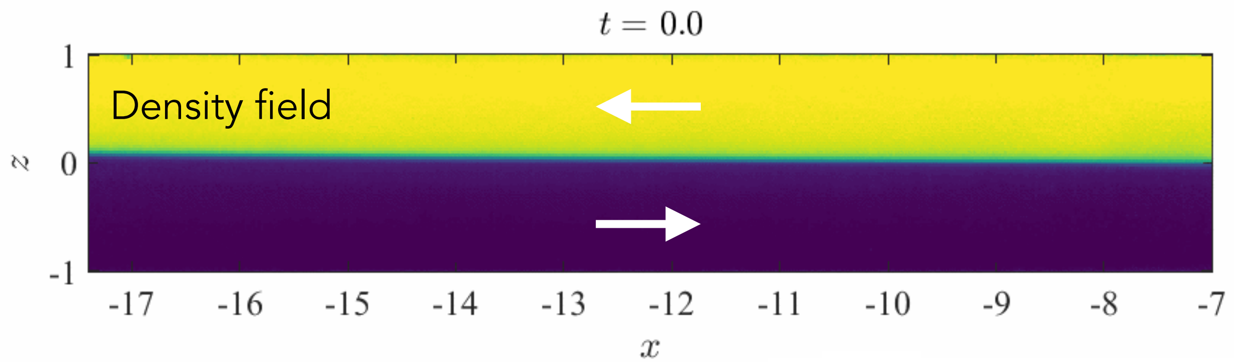

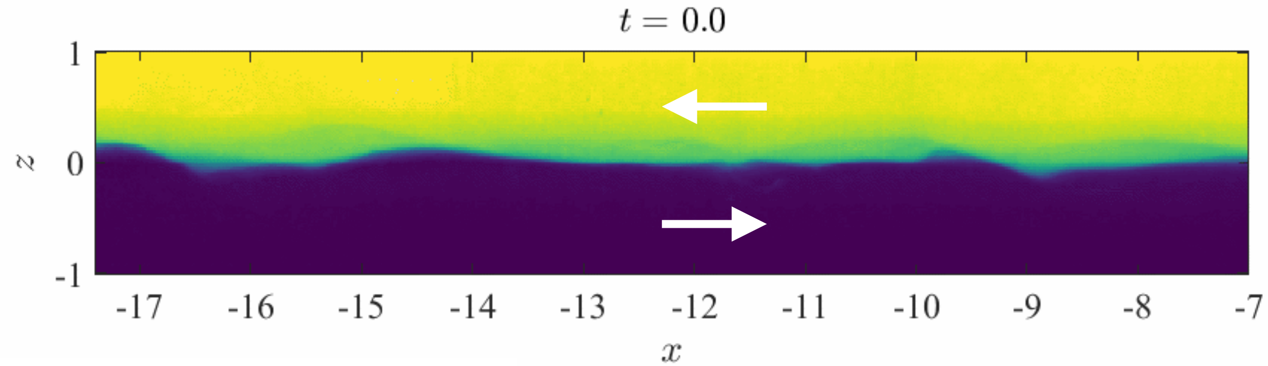

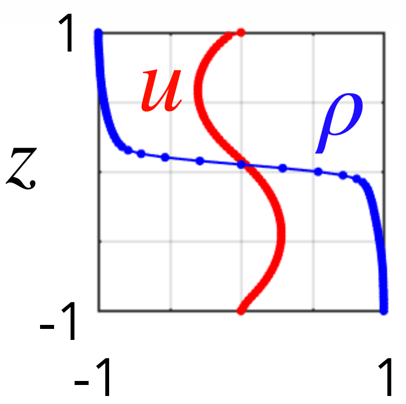

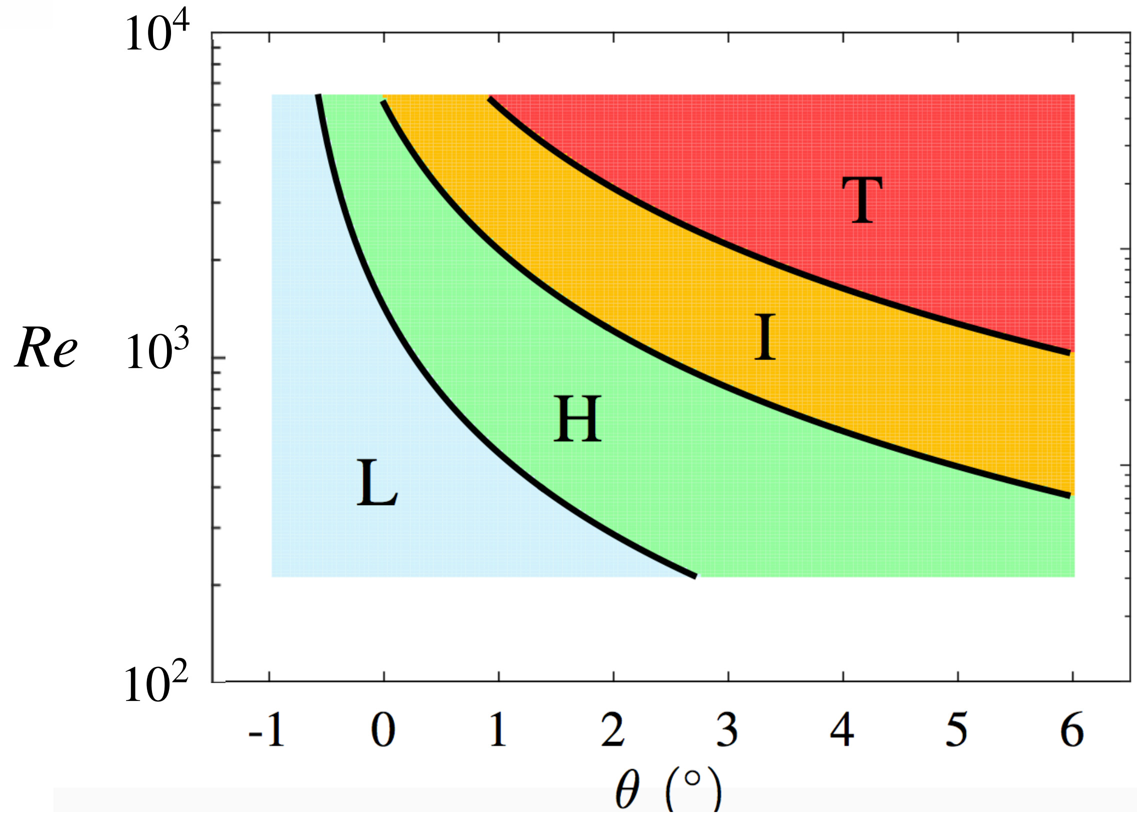

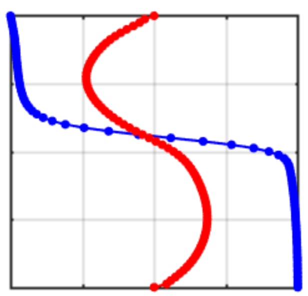

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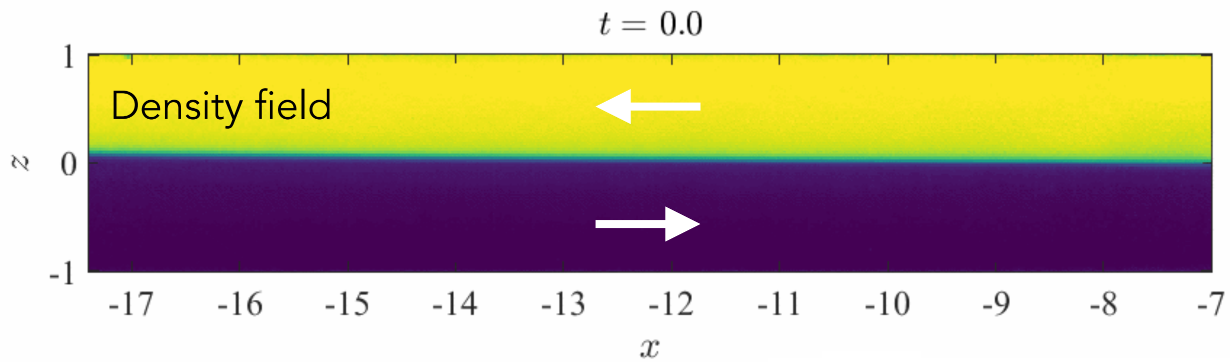
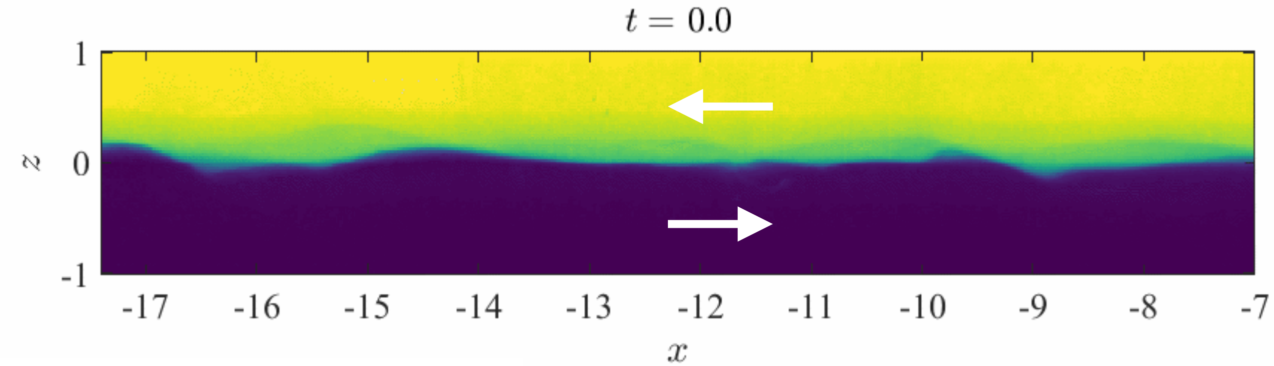
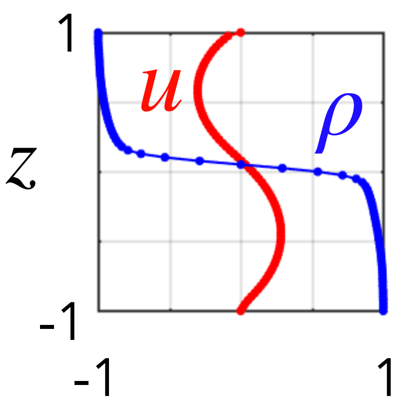
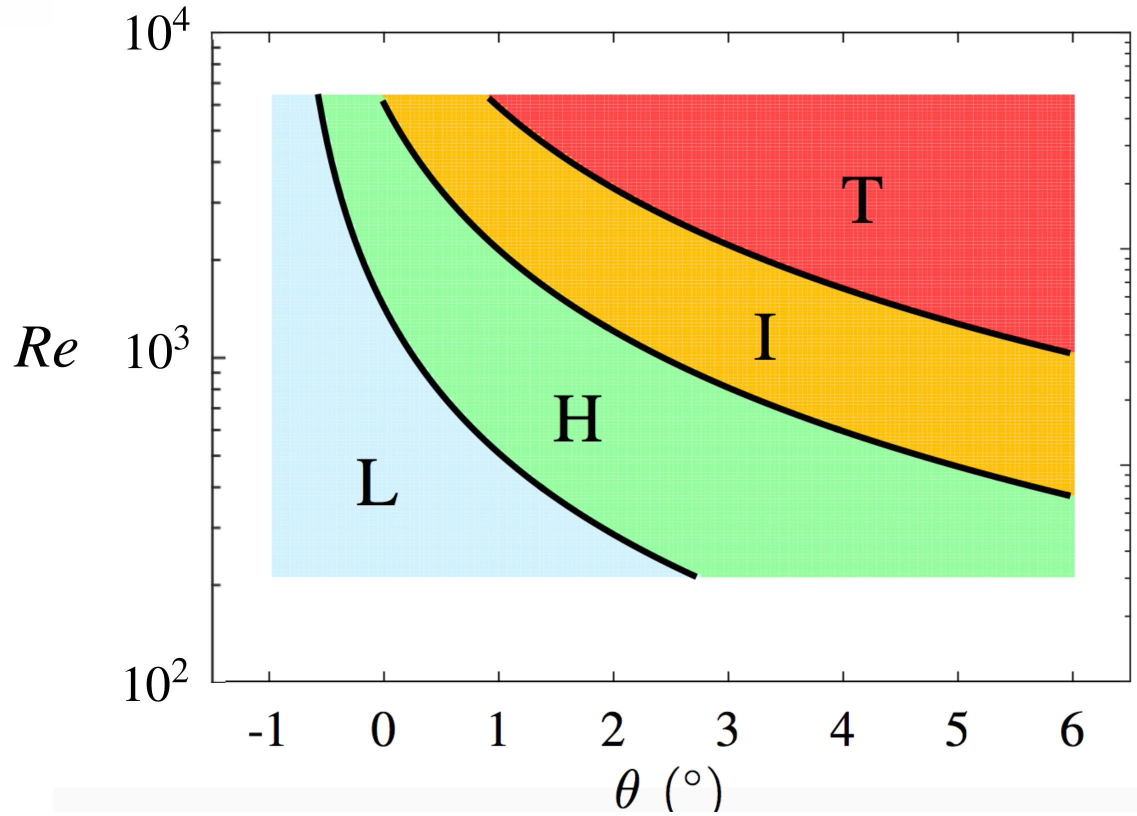
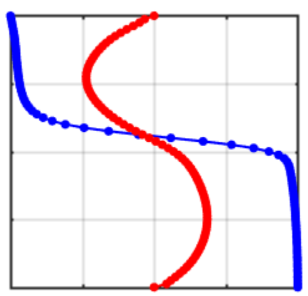
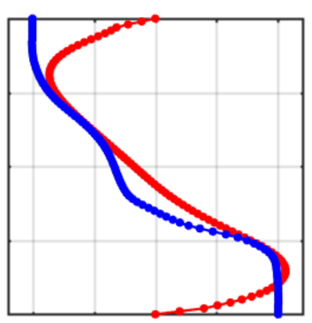




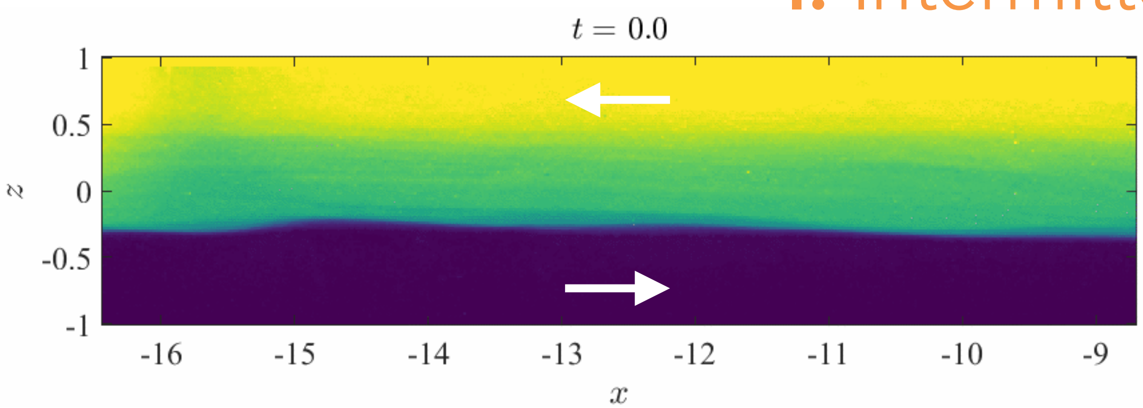








I: Intermittent



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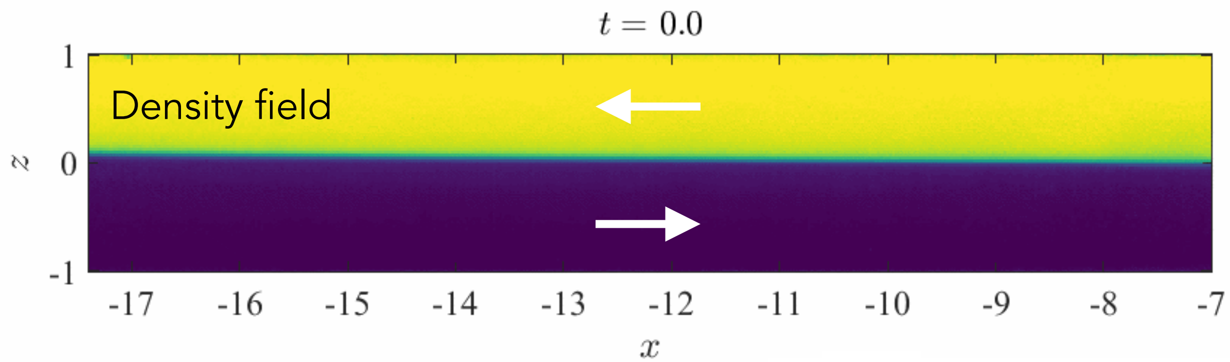

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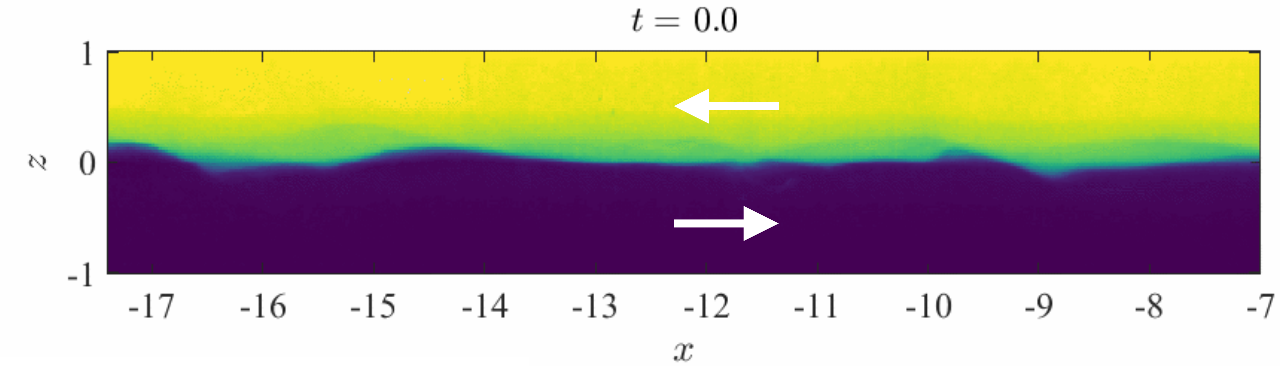

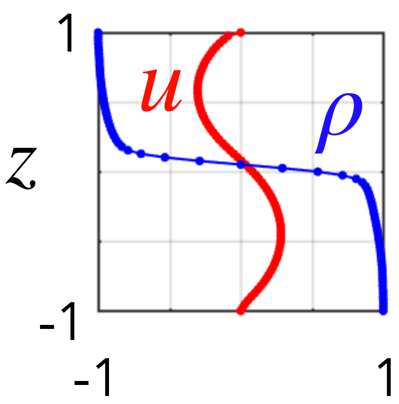

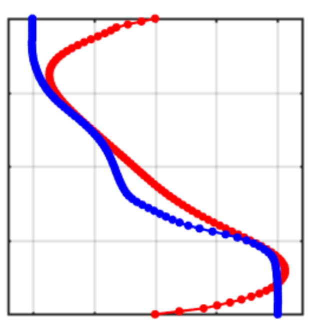

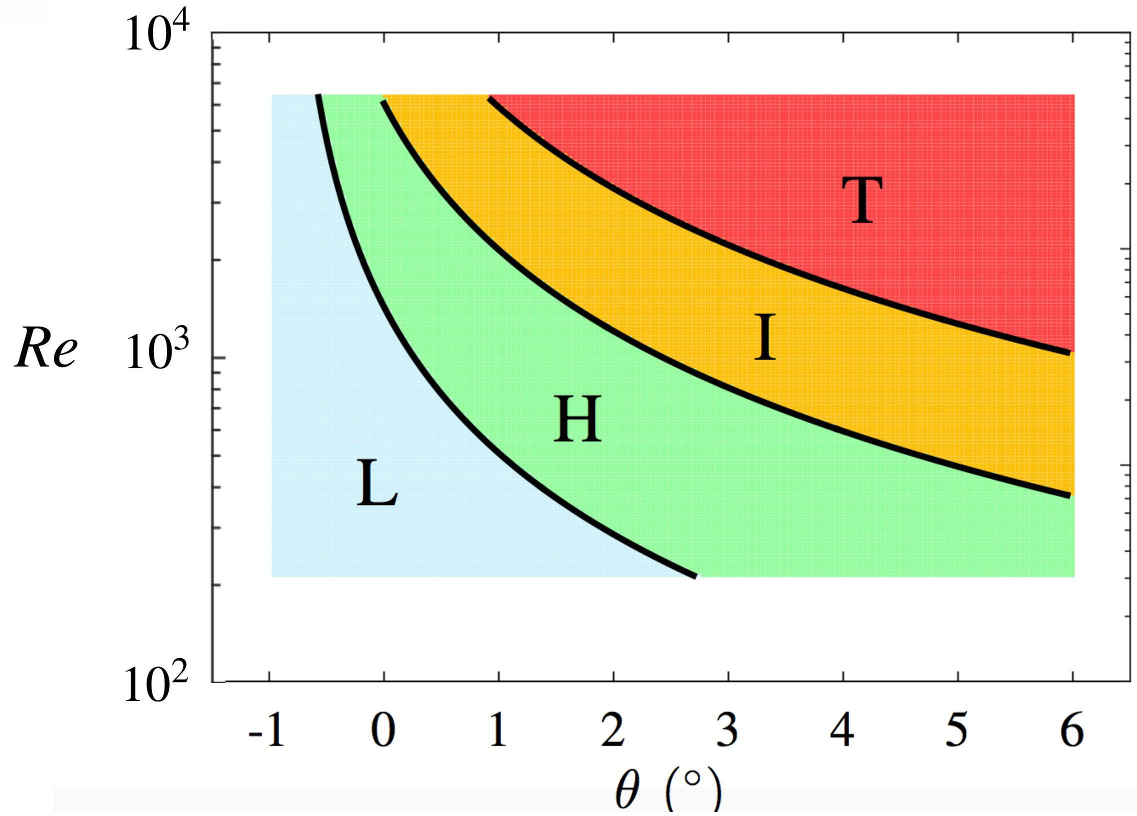

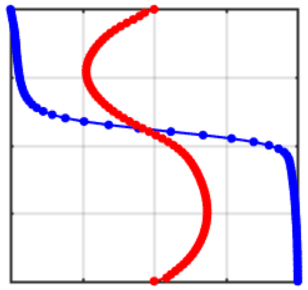

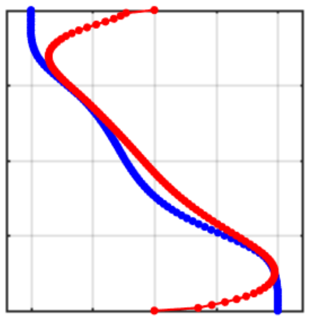

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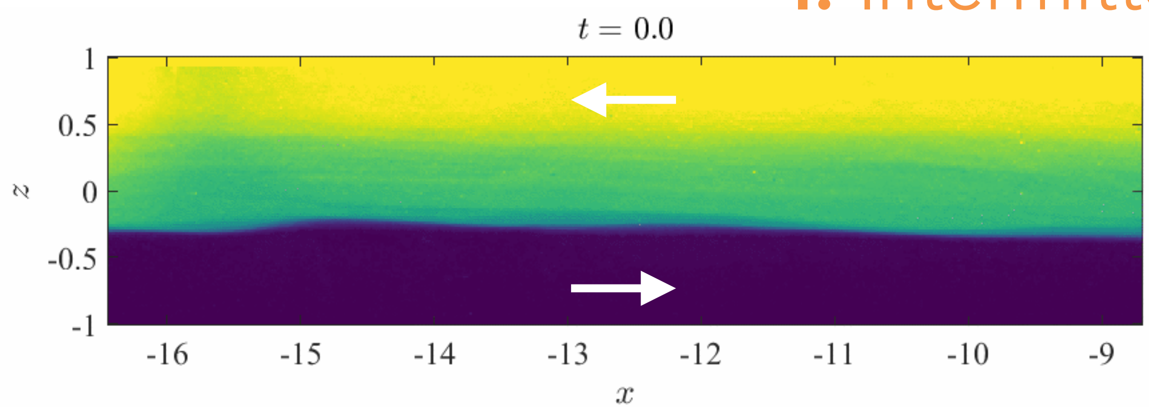
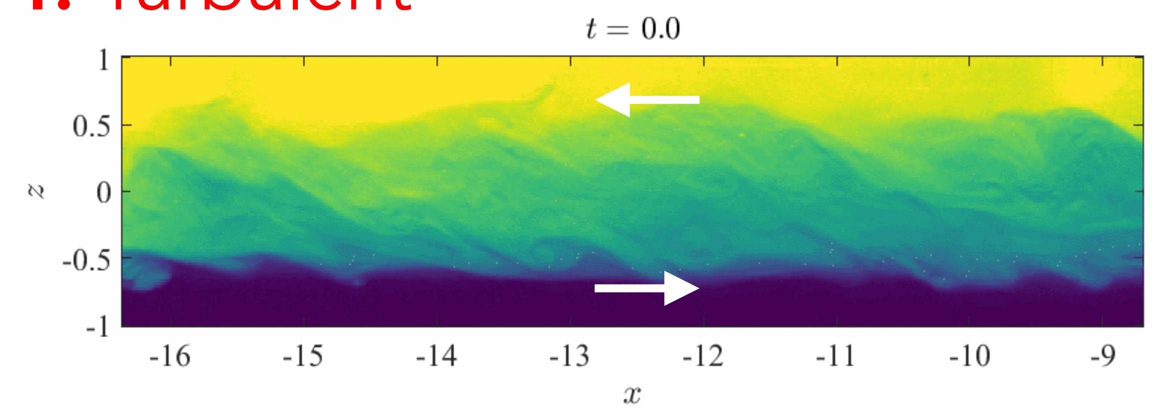
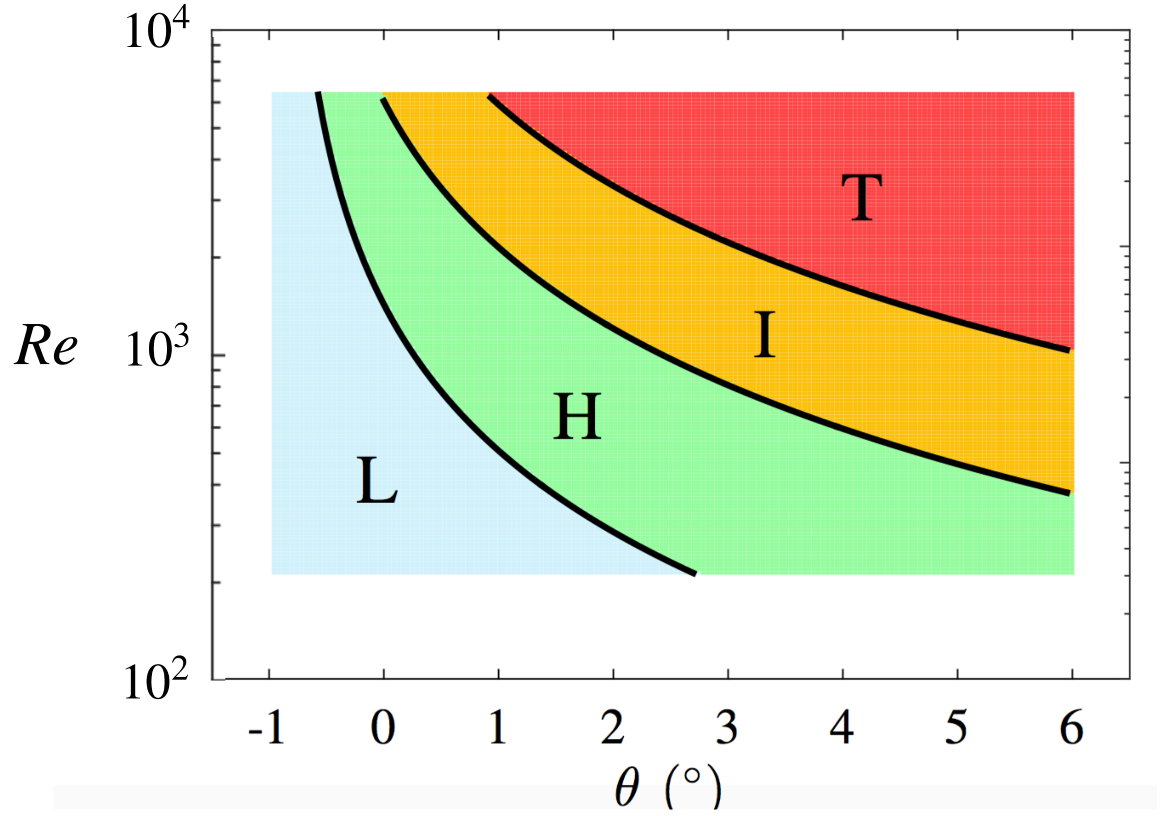


T: Turbulent



Flow regimes in the SID



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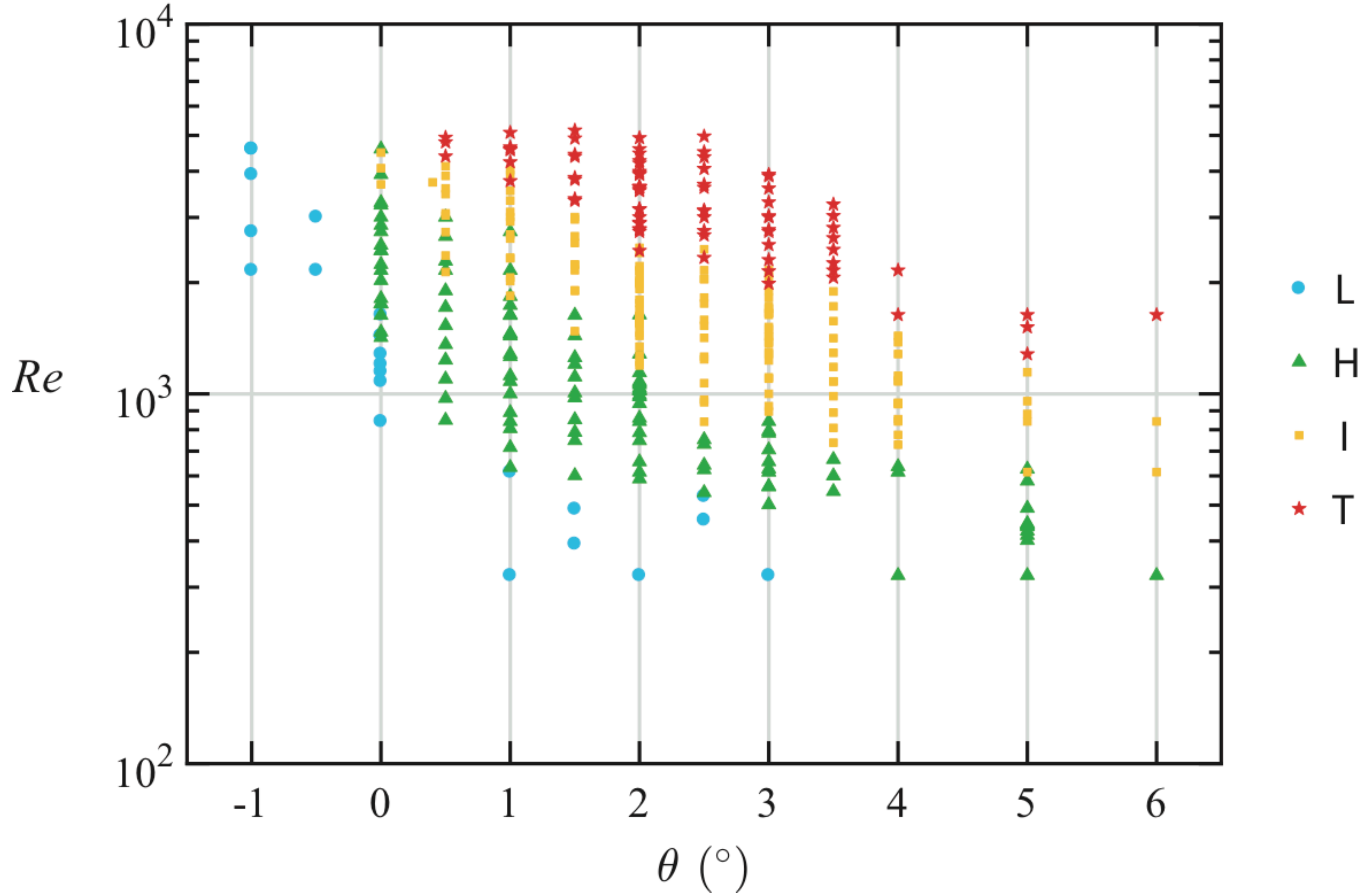

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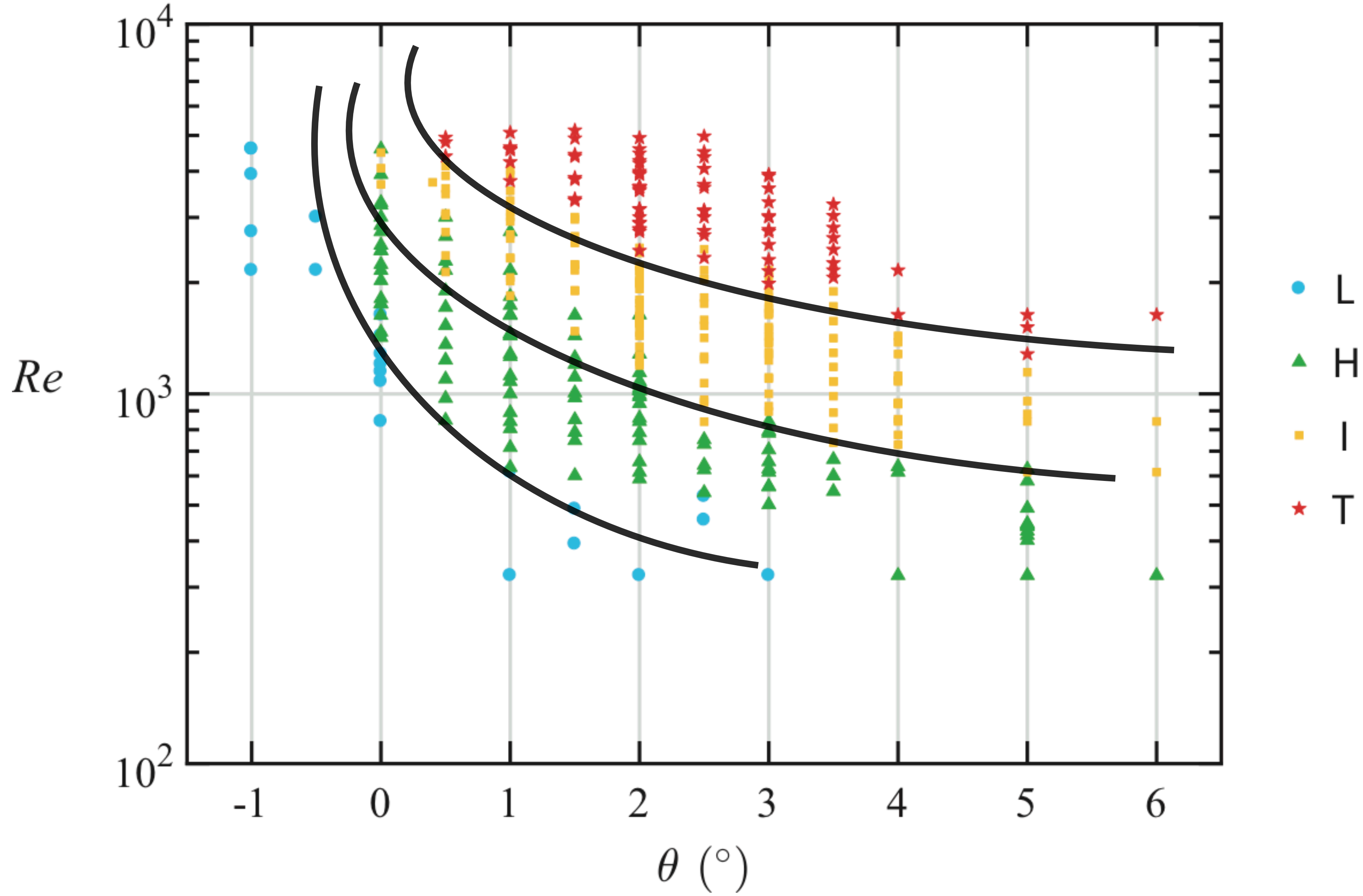
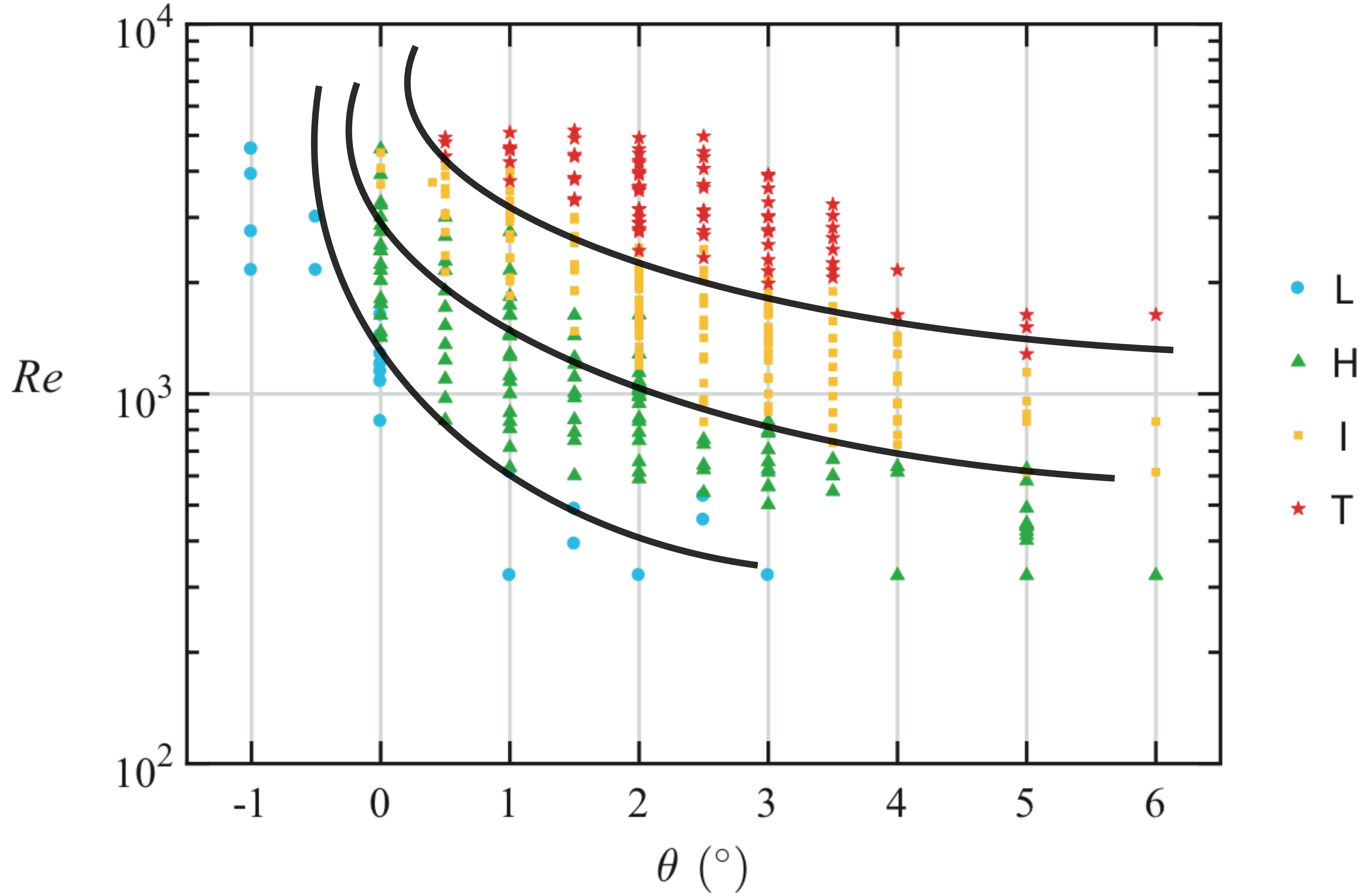


Flow regimes in the SID

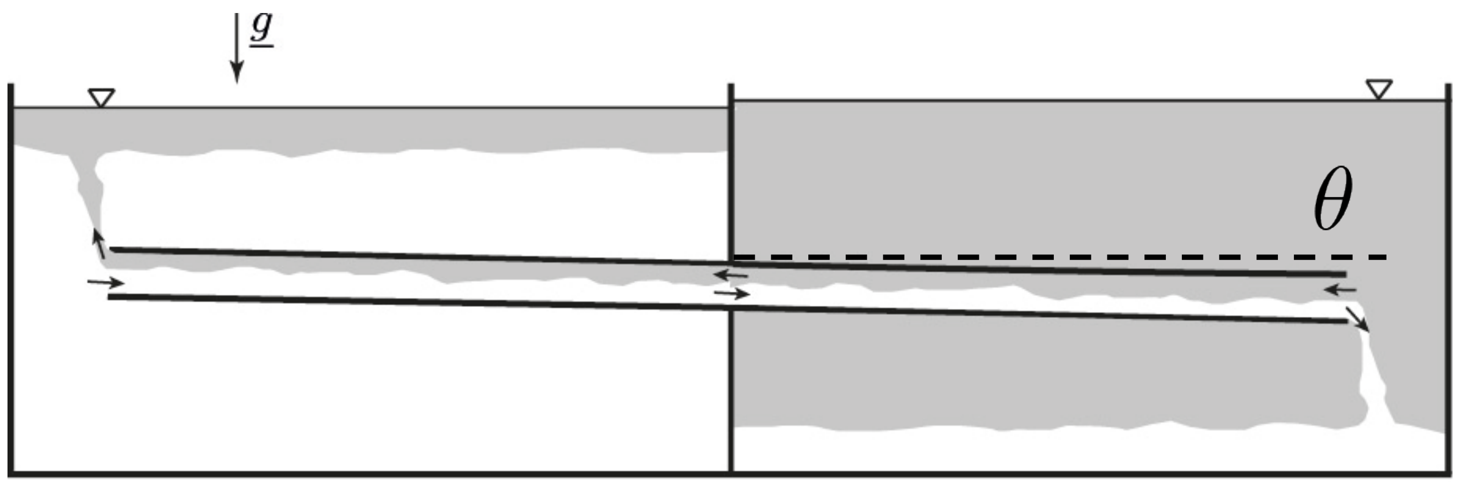
In this talk:

How do these **regime transitions scale with θ, Re ?**



Previous work and insight

Meyer & Linden (2014)



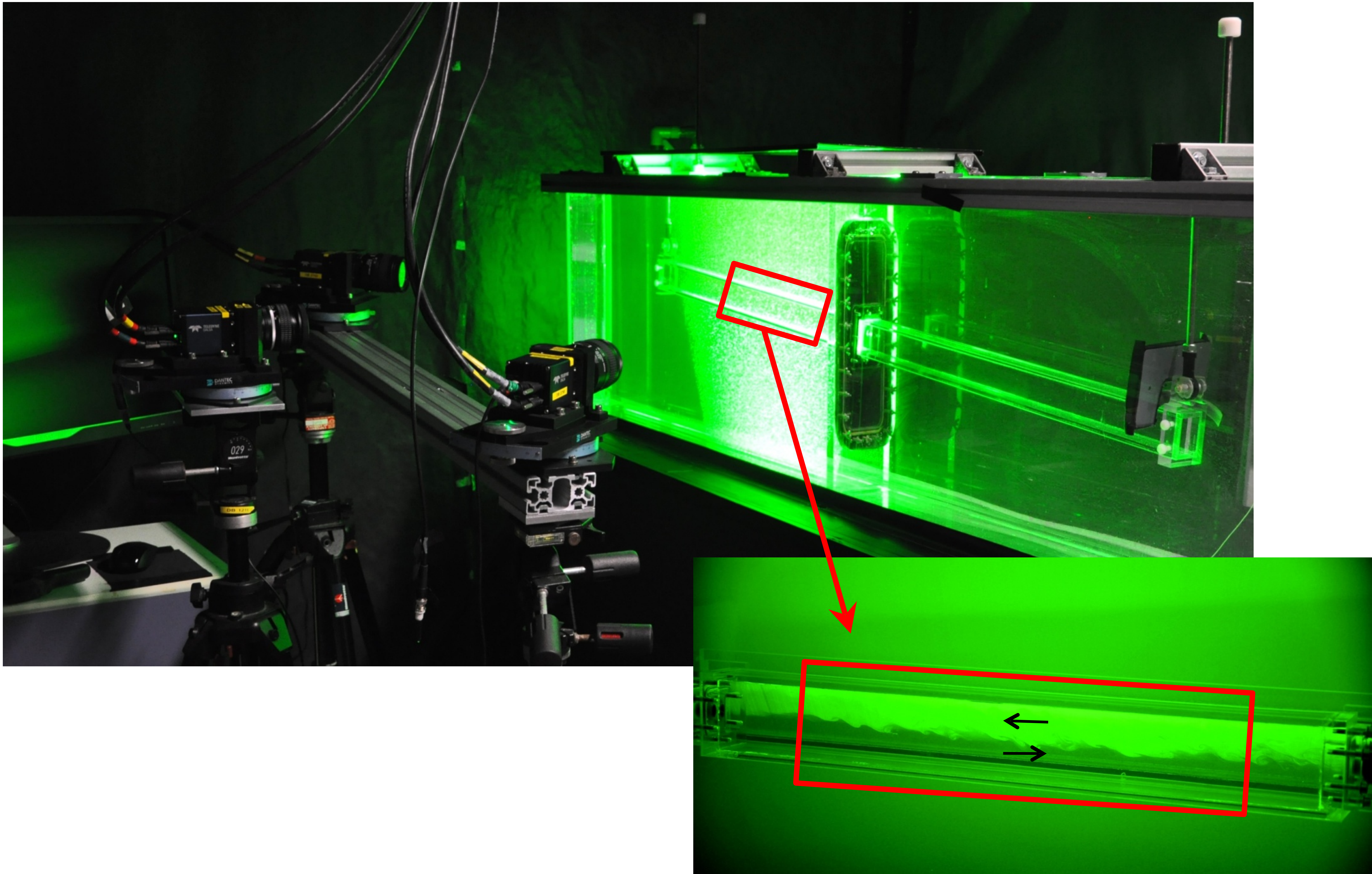
- Two-layer **hydraulic control**: critical Froude number: $(\Delta U)^2 \sim g'H$
- **Extra kinetic energy** from acceleration along duct: $(\Delta U_+)^2 \sim g'L\sin\theta$ **must be dissipated turbulently**

$\longrightarrow$ Non-dimensional scaling law?

$\longrightarrow$ Experimental confirmation?

Volumetric measurements

Partridge, Lefauve & Dalziel (2019)



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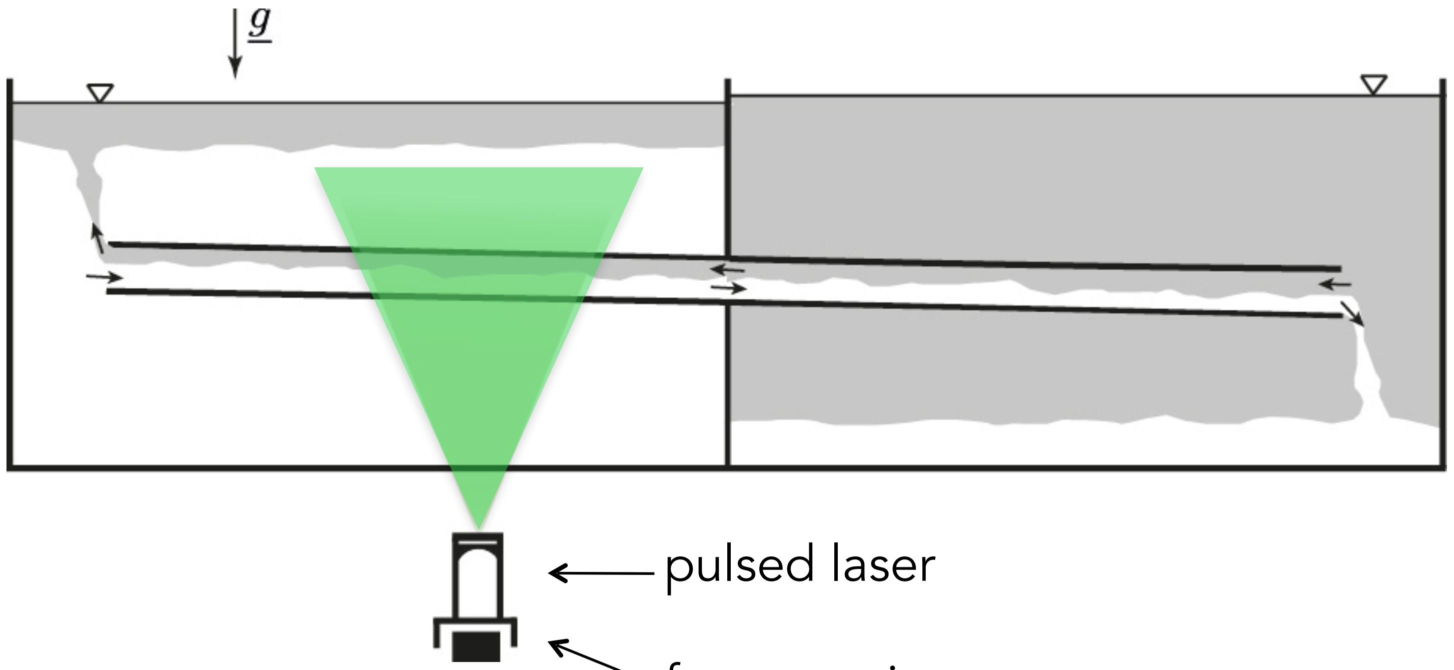
xz plane



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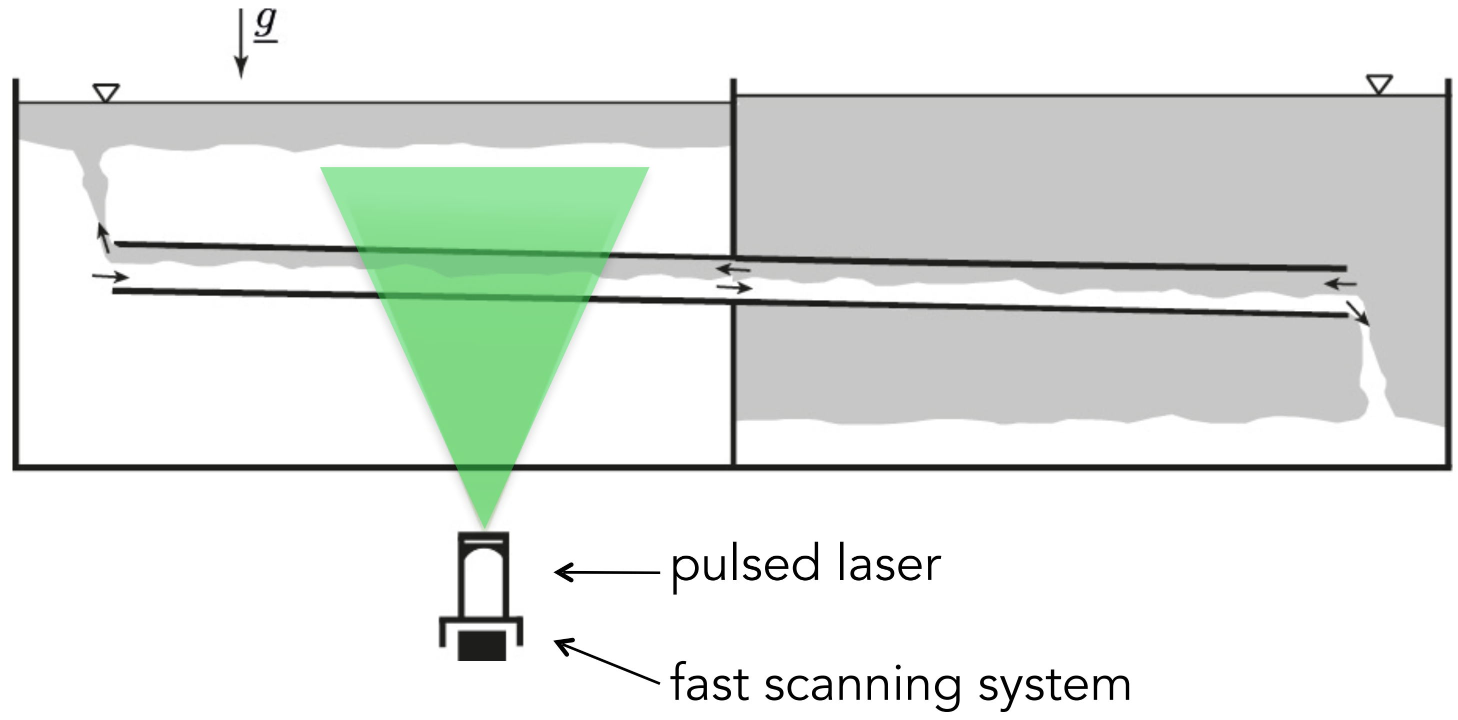

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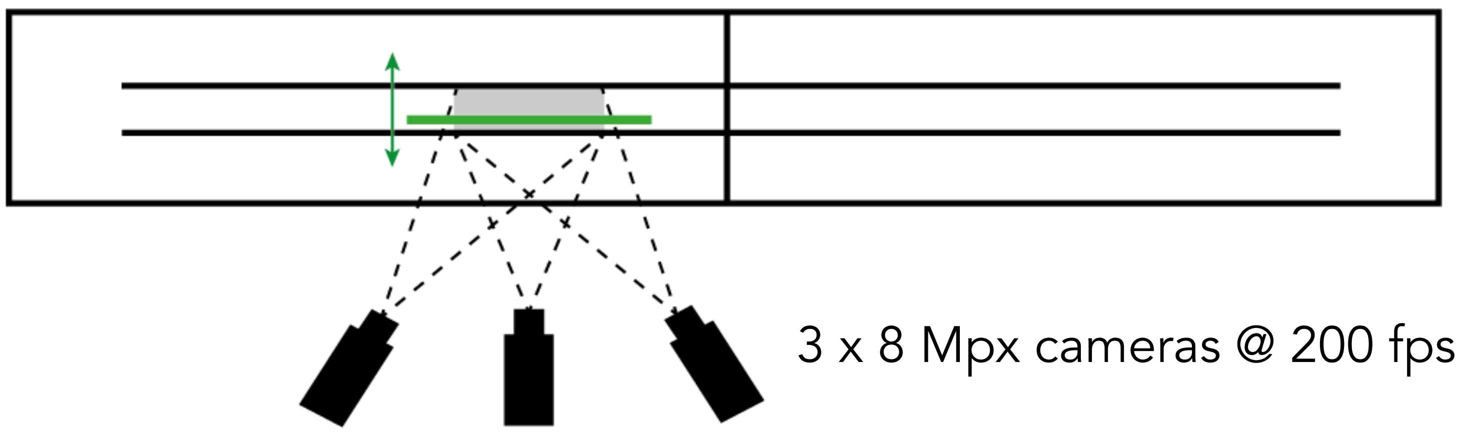

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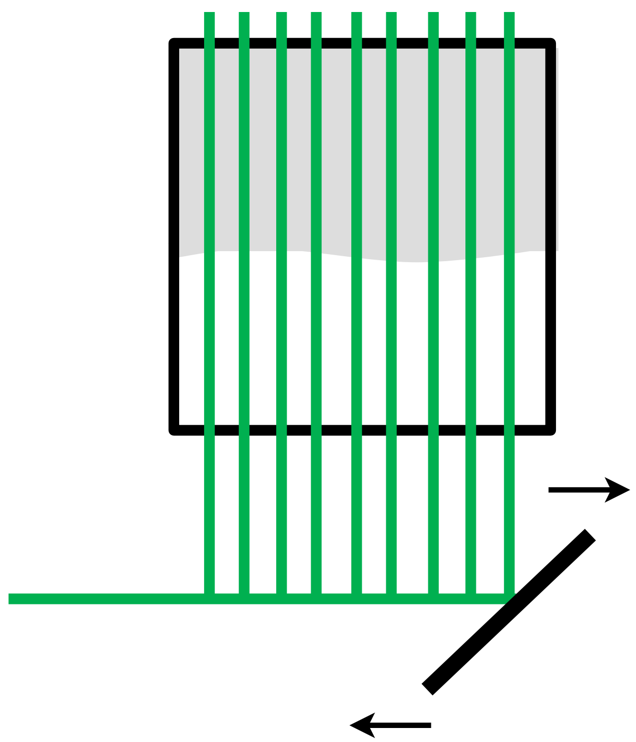

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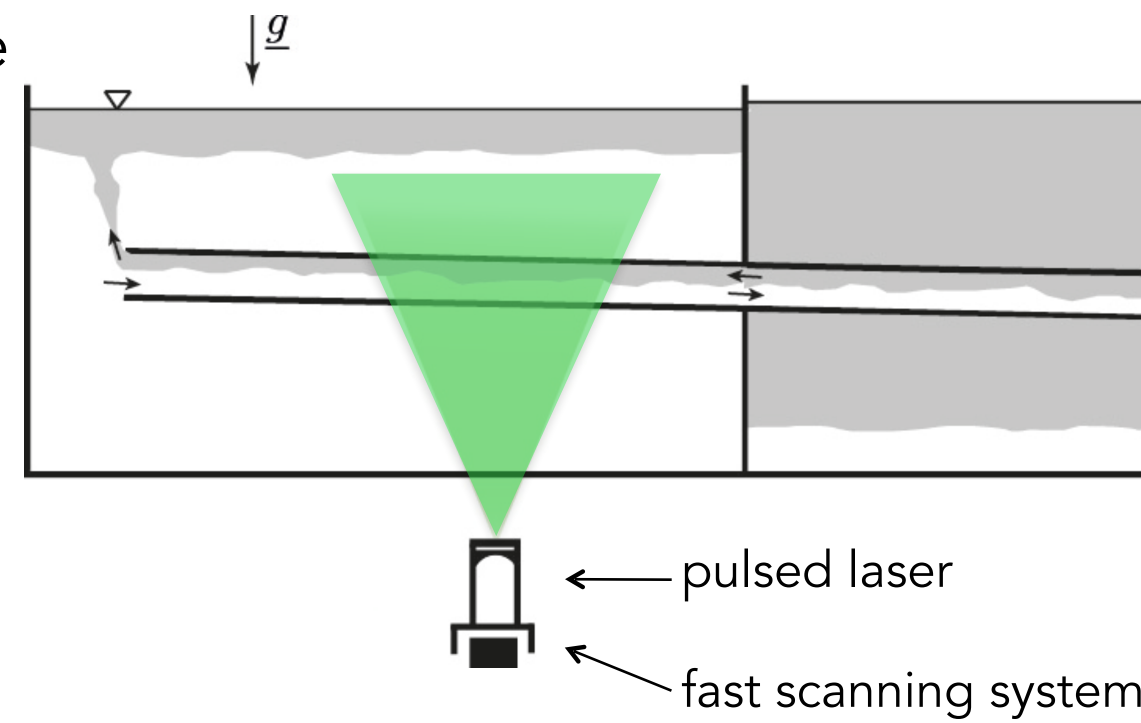

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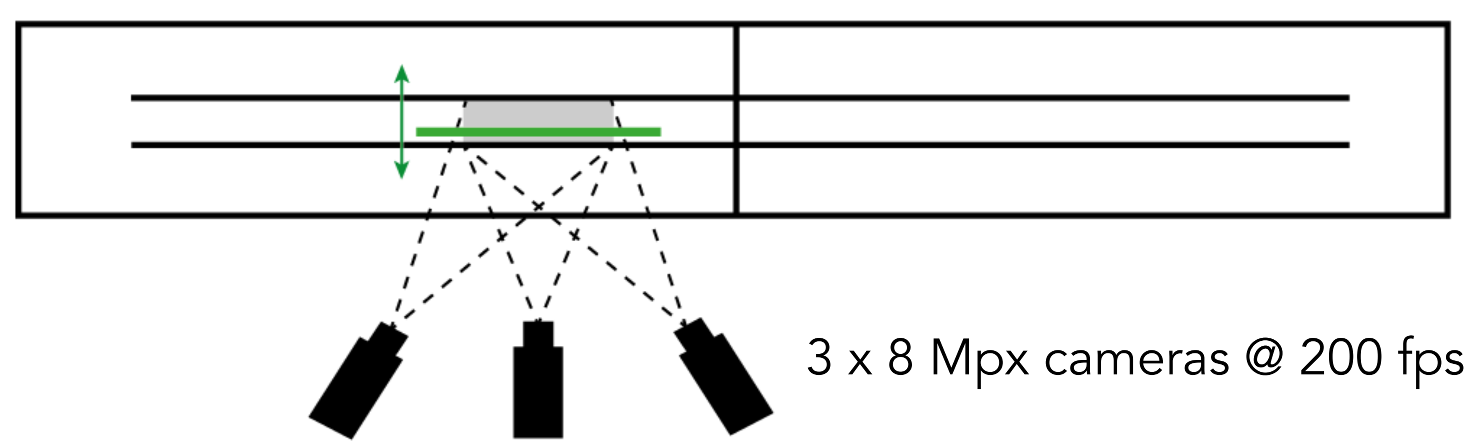

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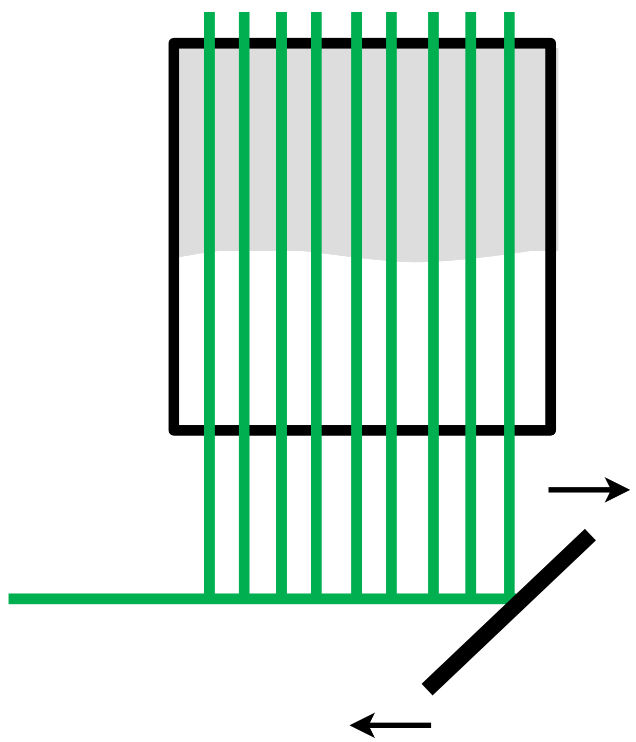

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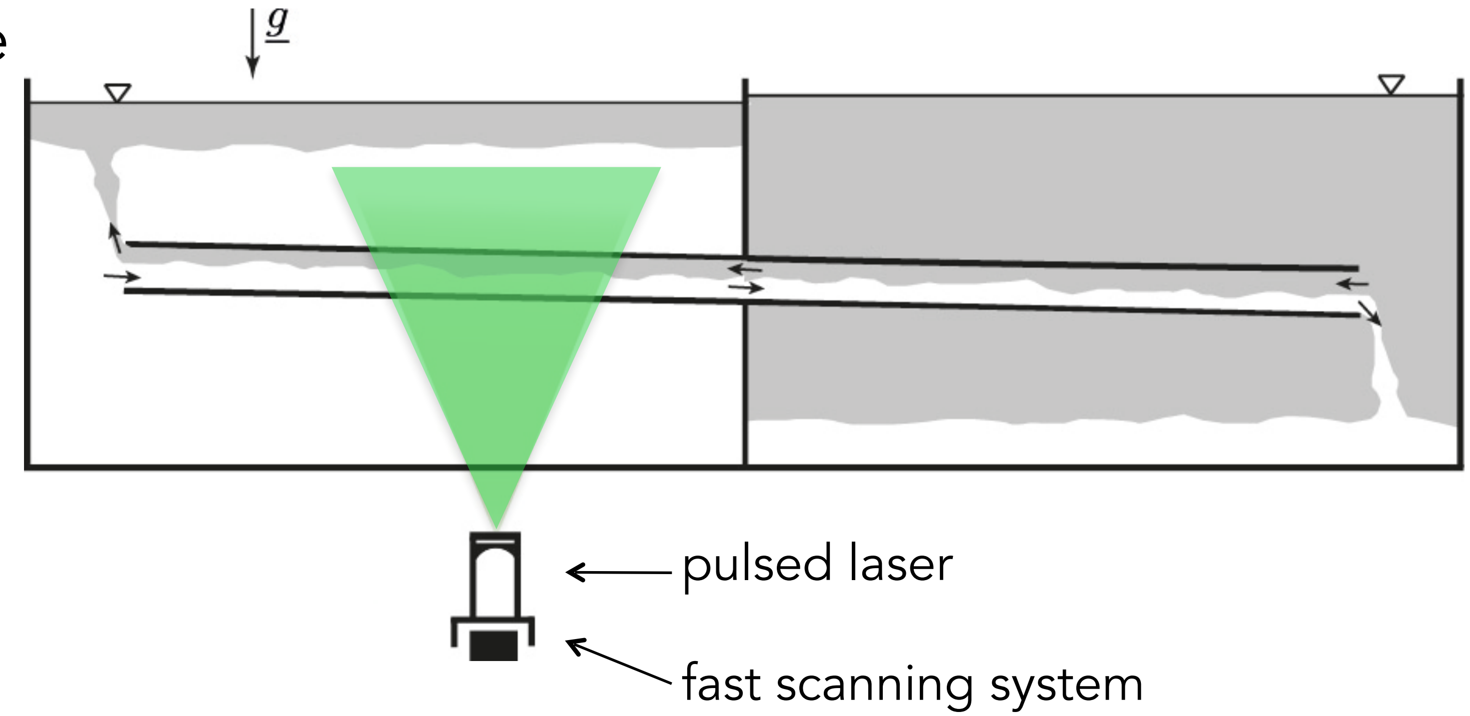

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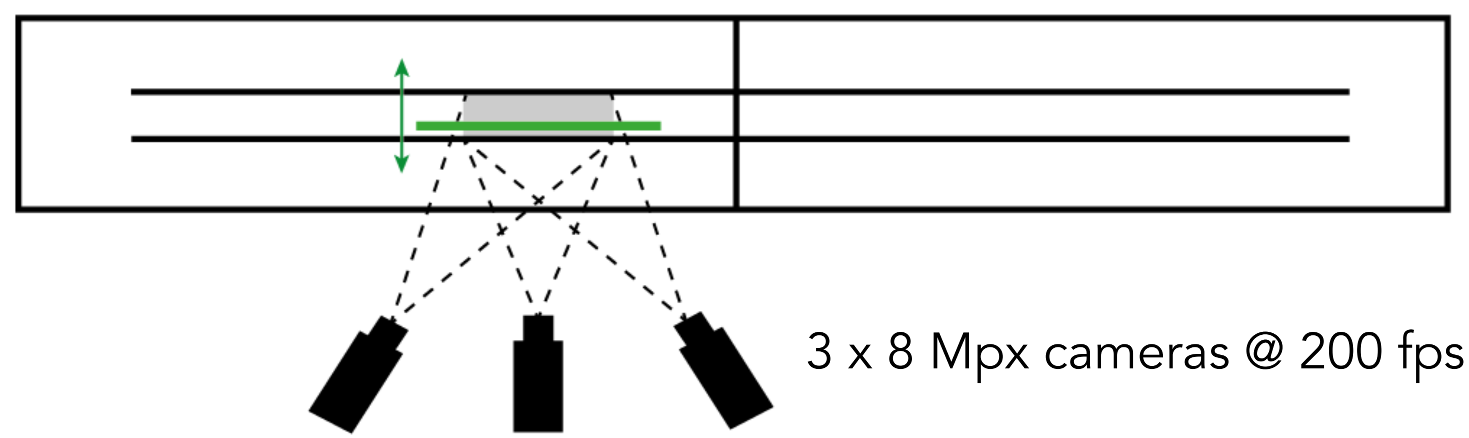

Stereo **P**article **I**mage **V**elocimetry

+

Planar **L**aser **I**nduced **F**luorescence

$$\longrightarrow u, v, w, \rho\,(x, y_i, z, t_i)$$

Volumetric measurements

Partridge, Lefauve & Dalziel (2019)

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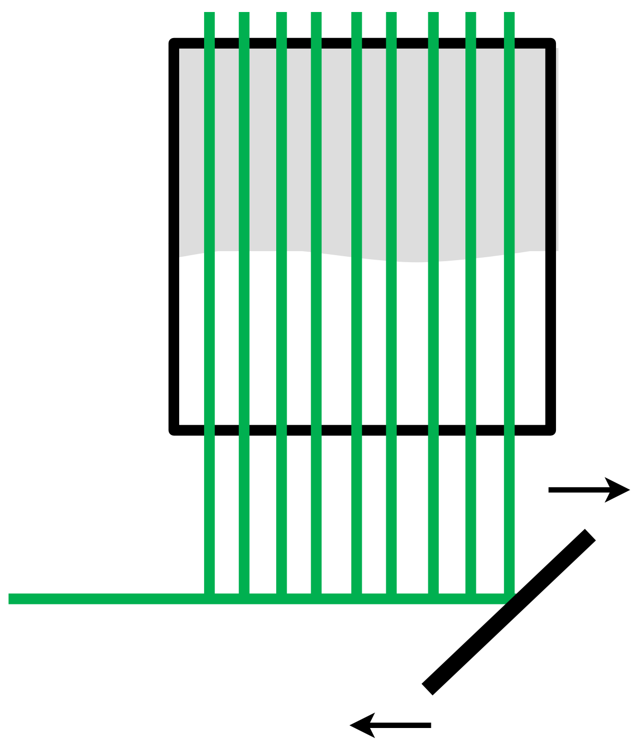

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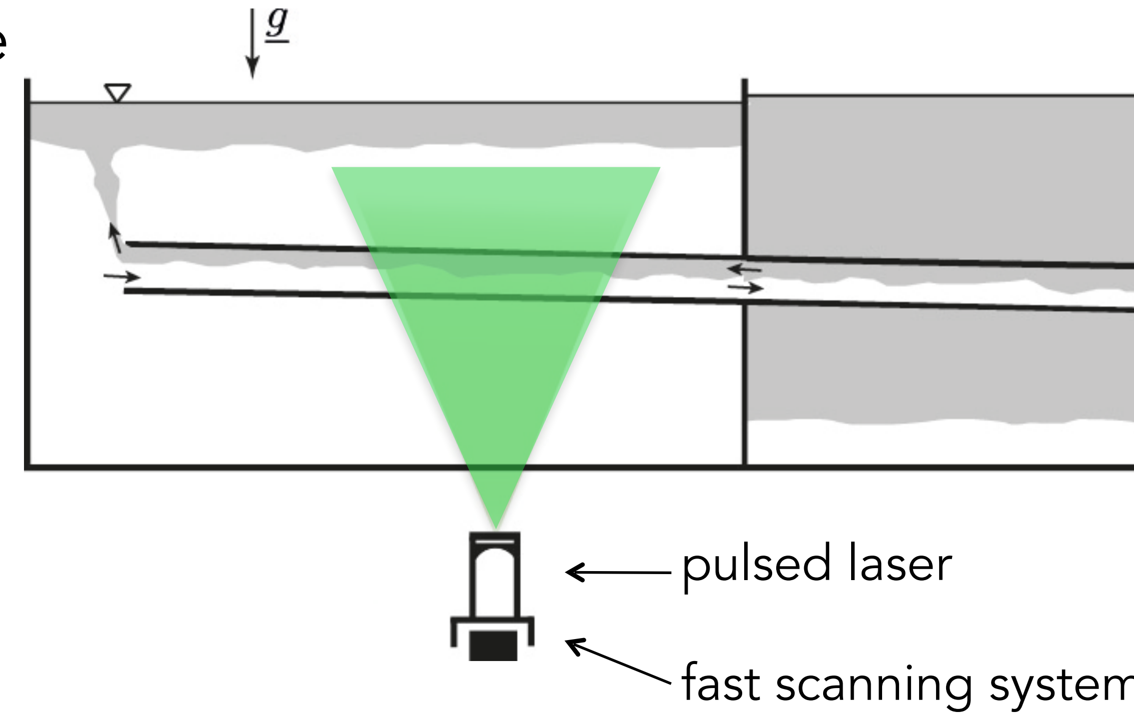

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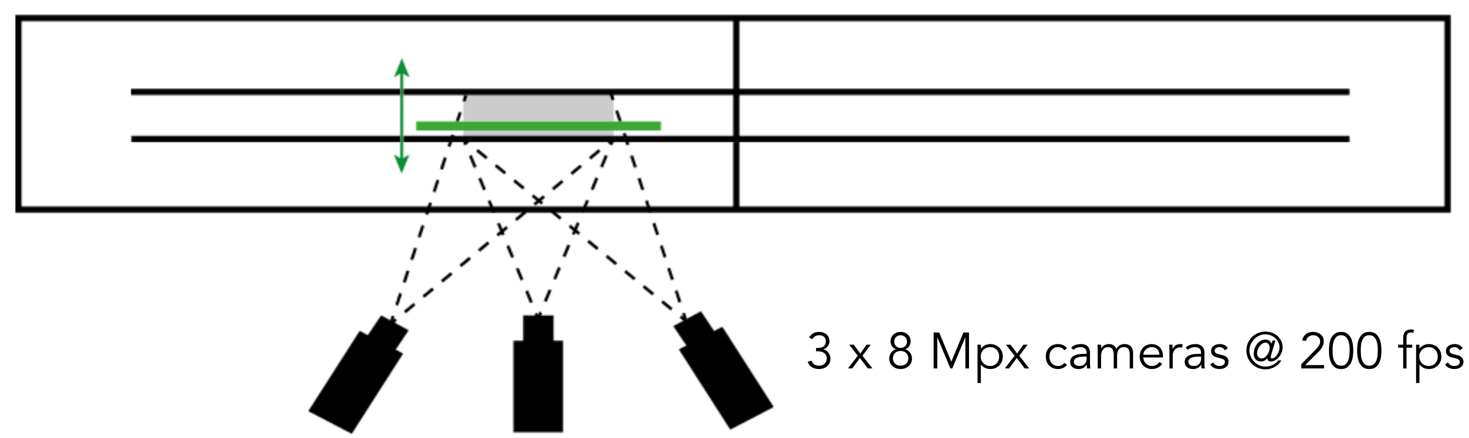

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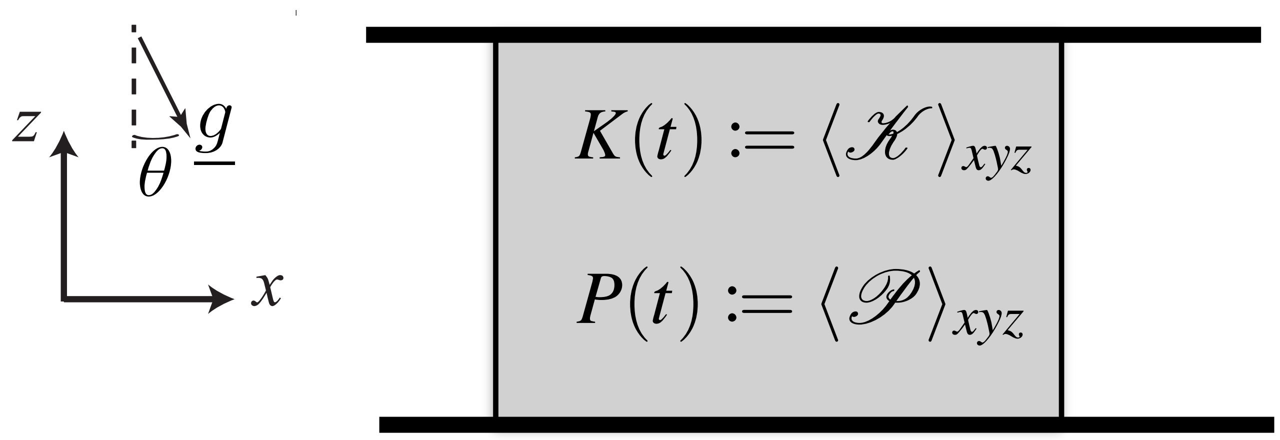
$\longrightarrow u, v, w, \rho\,(x, y_i, z, t_i)$

in $i = 1, \ldots, 30$ successive planes $\longrightarrow$ construct 3D volumes $u, v, w, \rho\,(x, y, z, t)$

vector yield ~ $4 \times 500 \times 30 \times 100 \times 300 \sim 2 \times 10^9$ / experiment

Energy budgets in control volume

Kinetic and potential energy **averaged in a control volume** V of the duct



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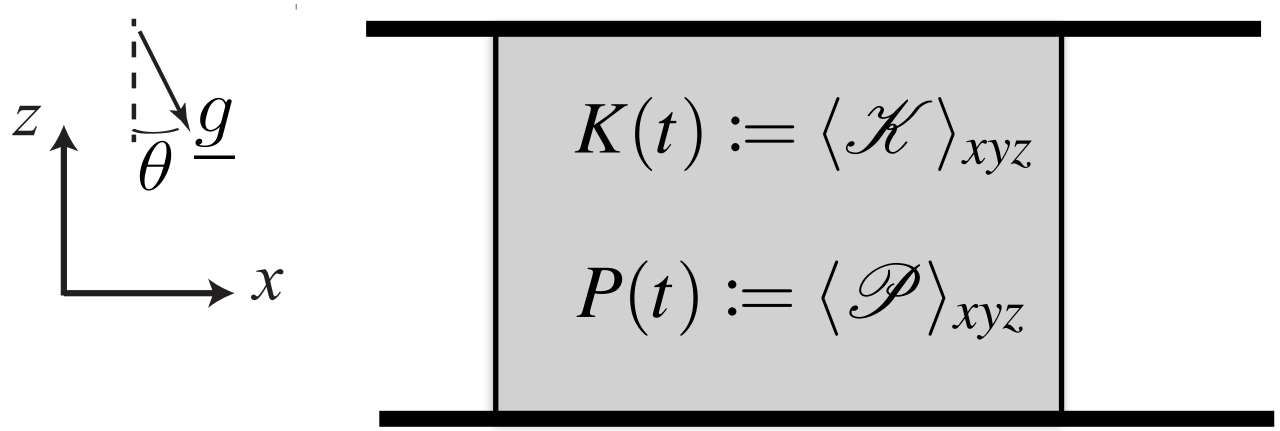

From first principles:

$$\frac{dK}{dt} = \Phi_K + B_x - B_z - D$$

$$\frac{dP}{dt} = \Phi_P - B_x + B_z,$$

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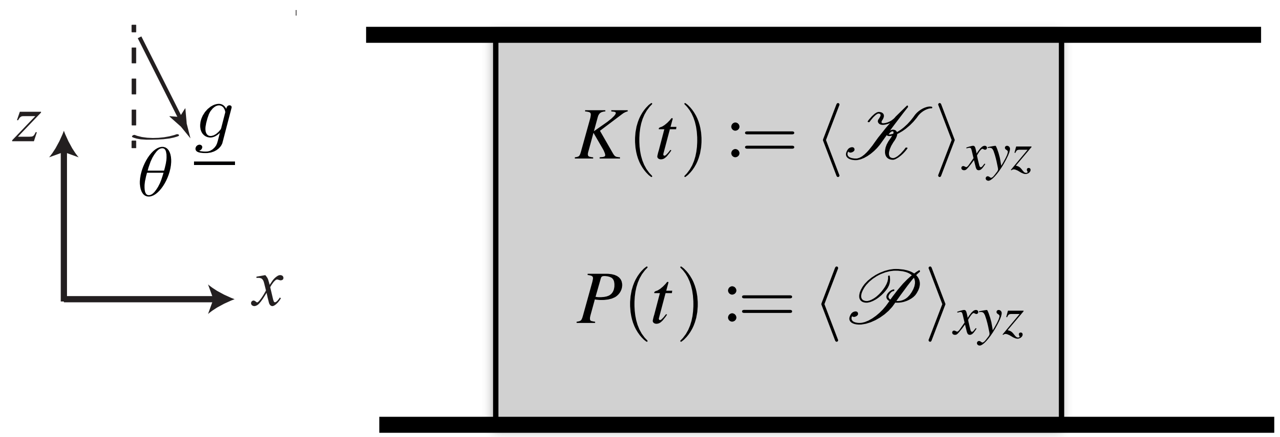

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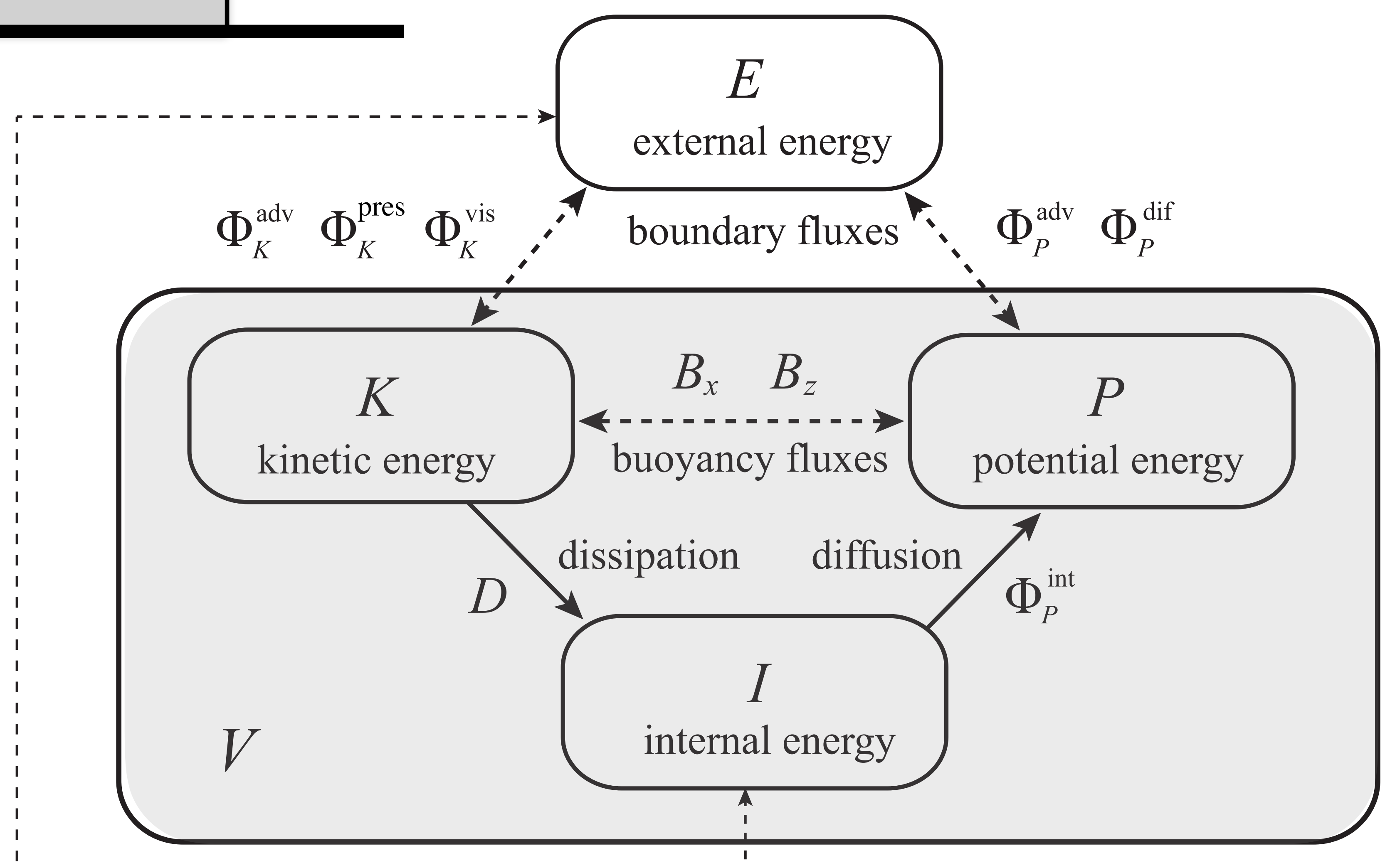
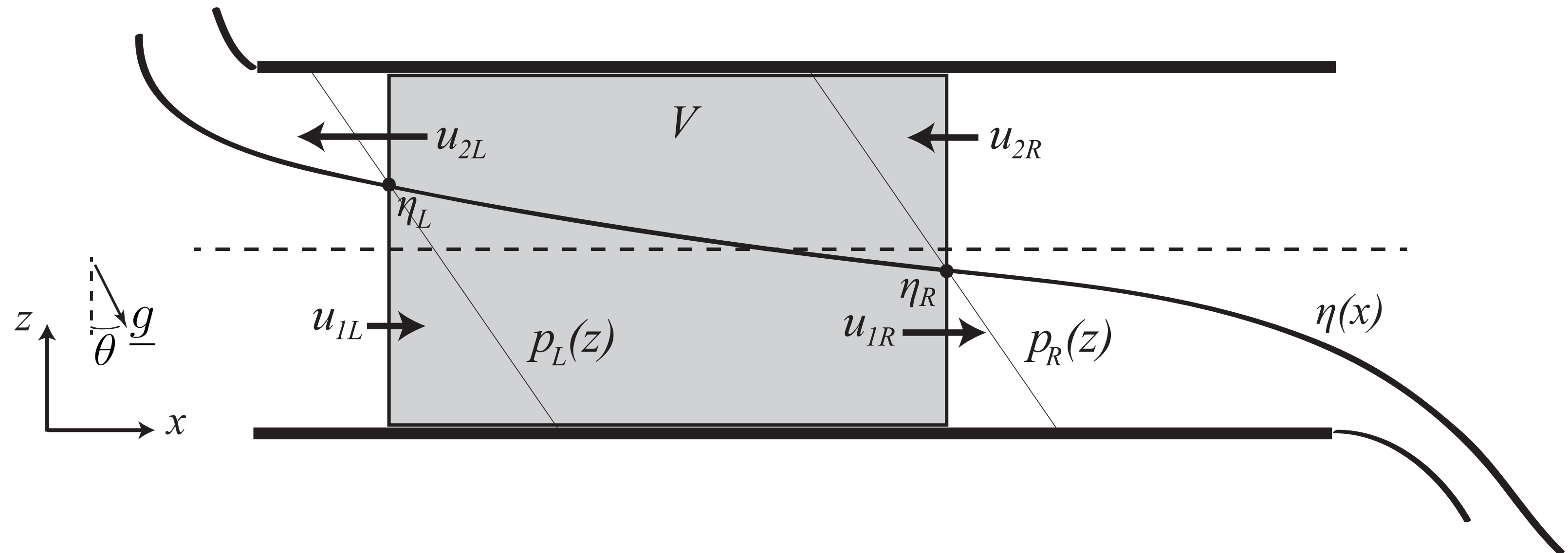
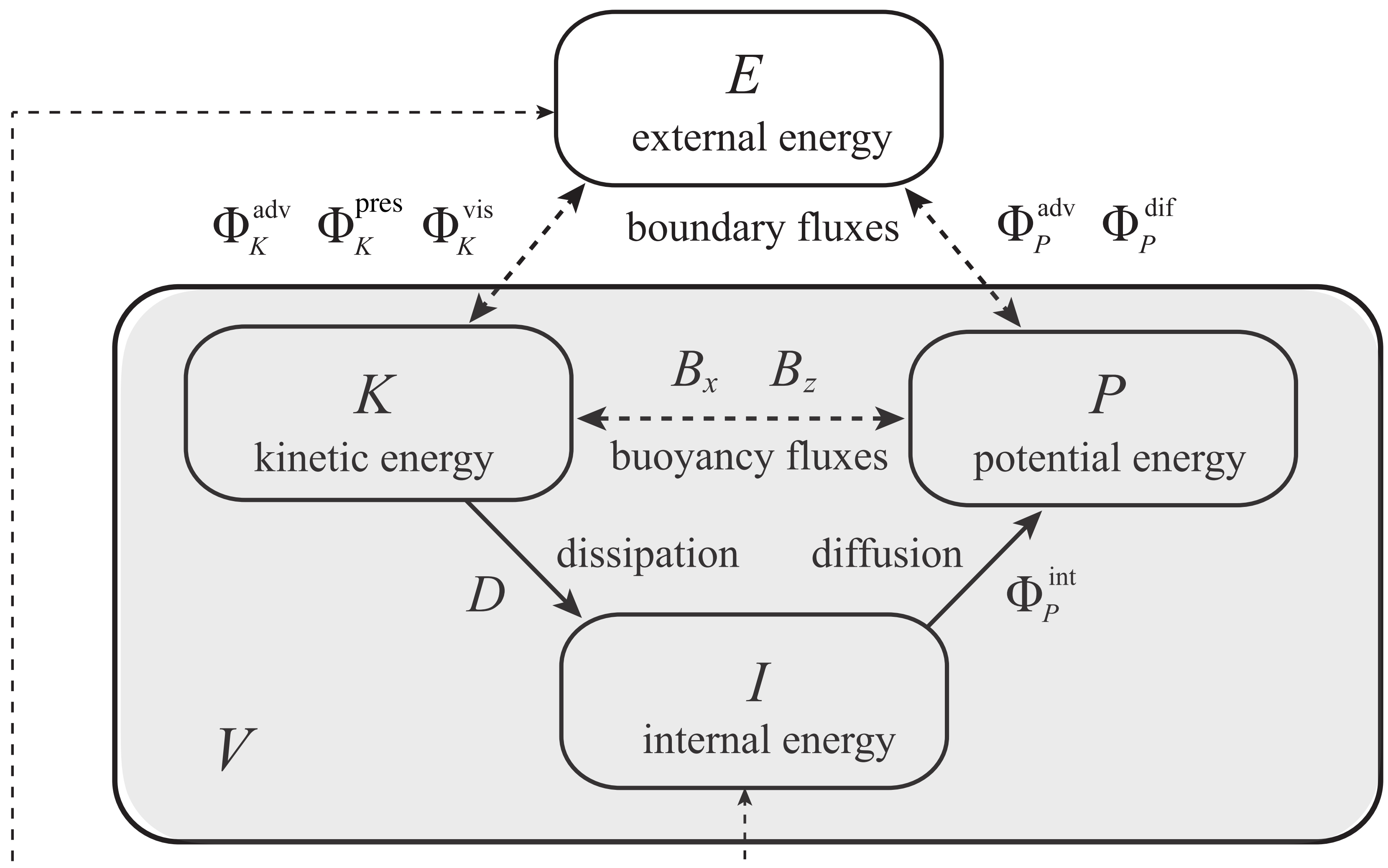


- Two-layer, near-hydrostatic
- High Re and Pr





Simplification #1

- Two-layer, near-hydrostatic
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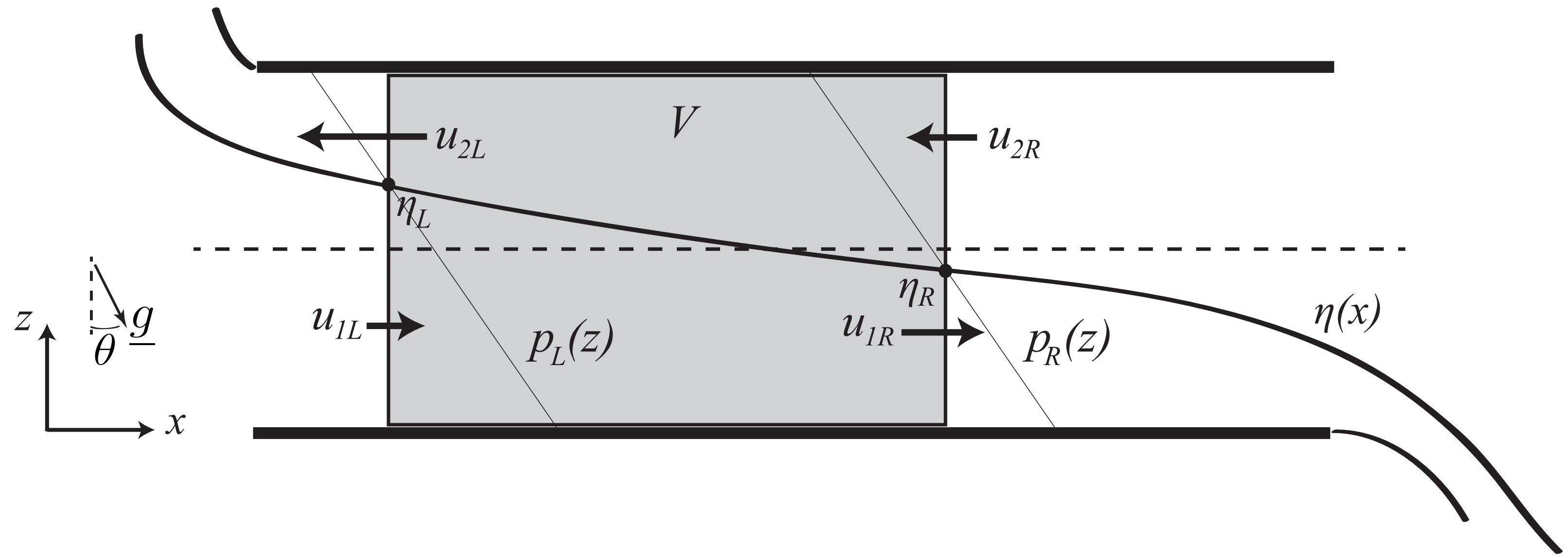

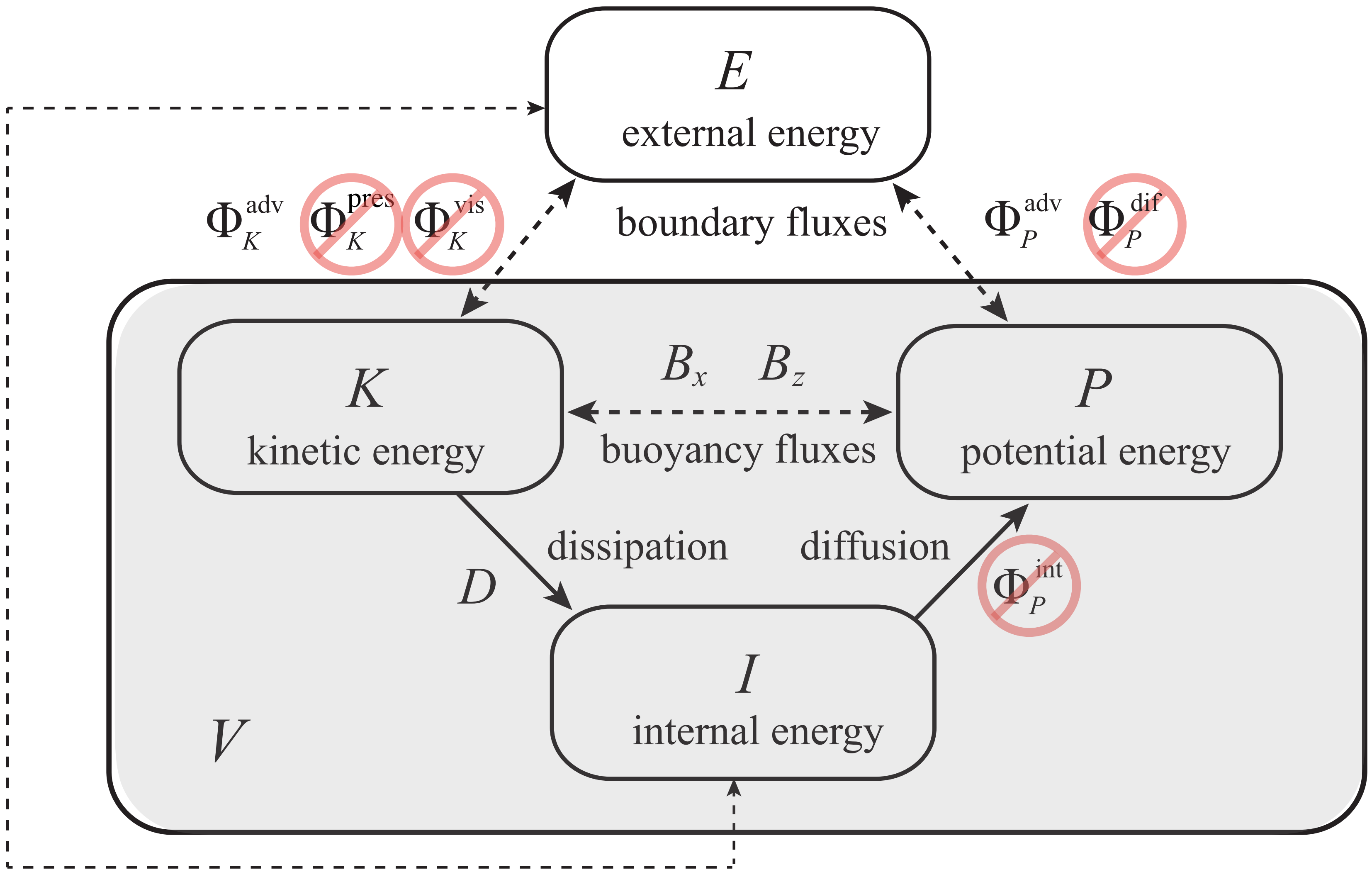

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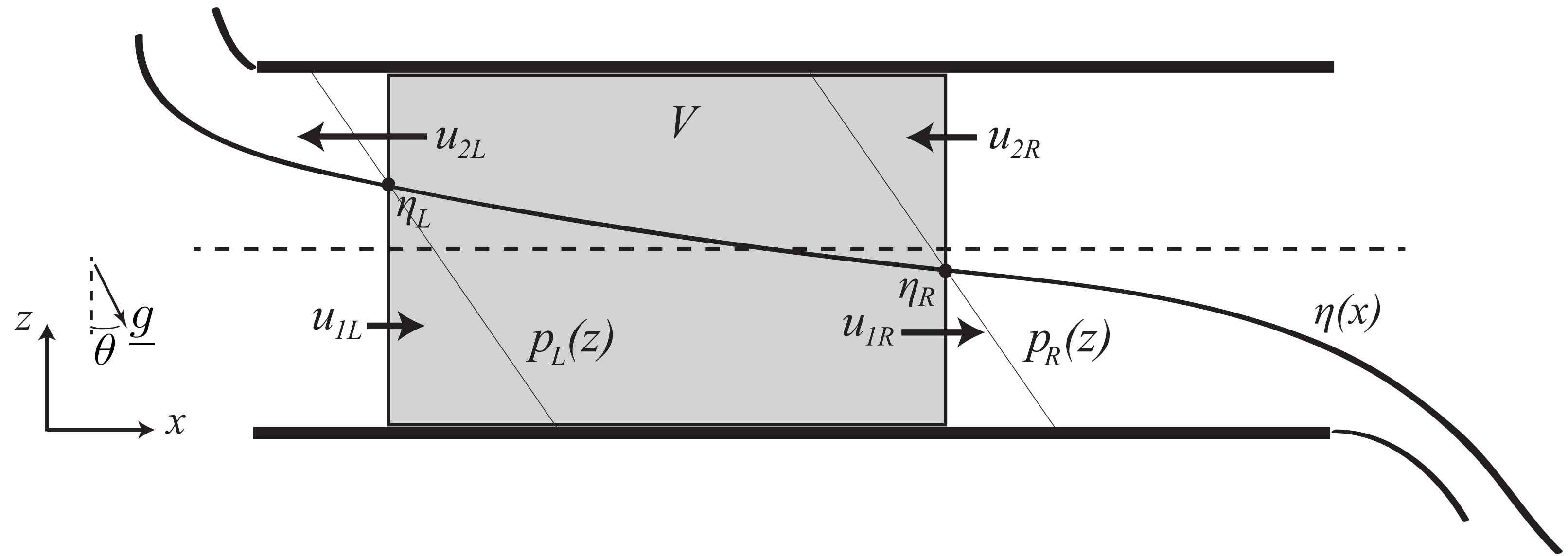

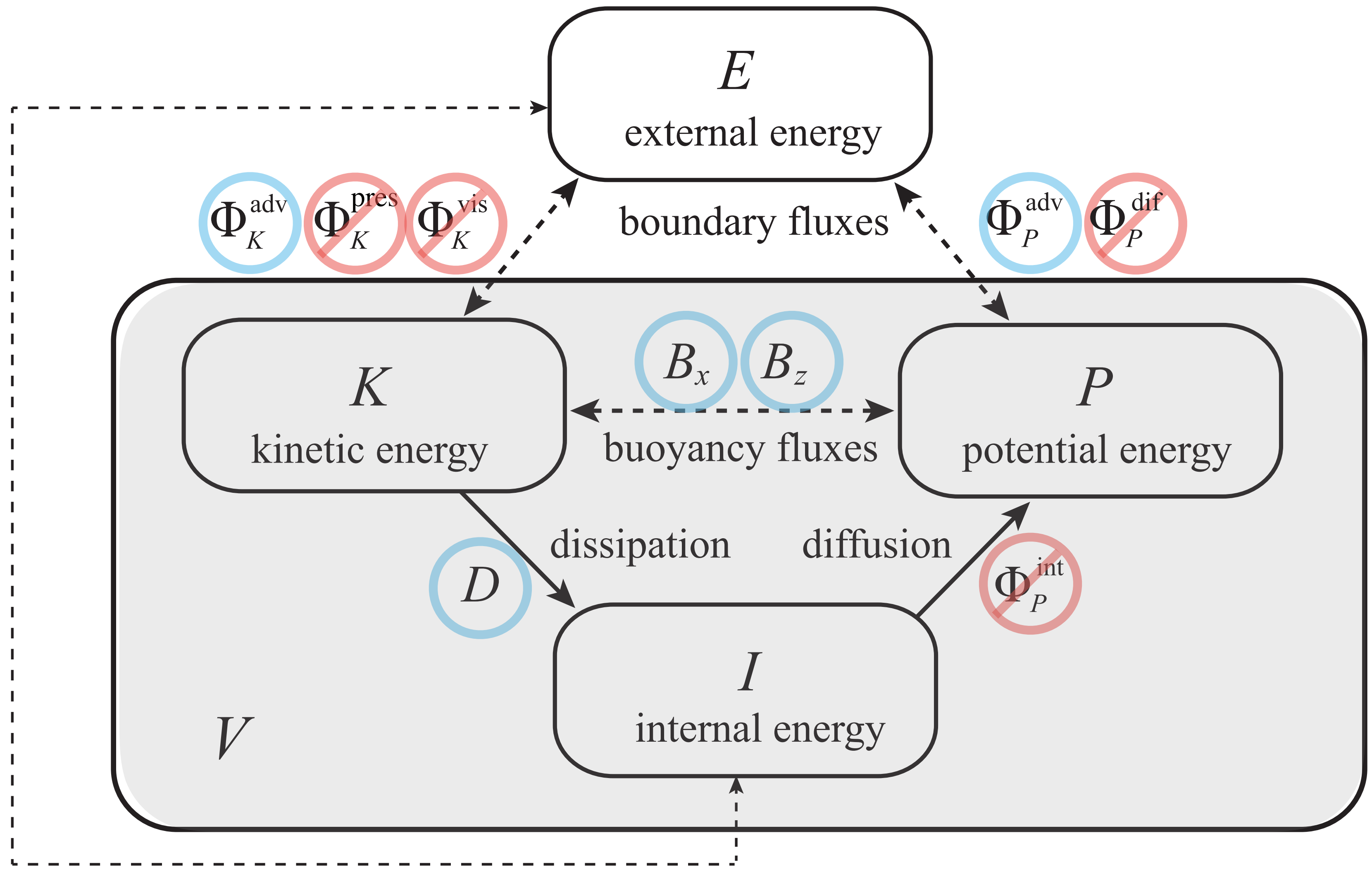

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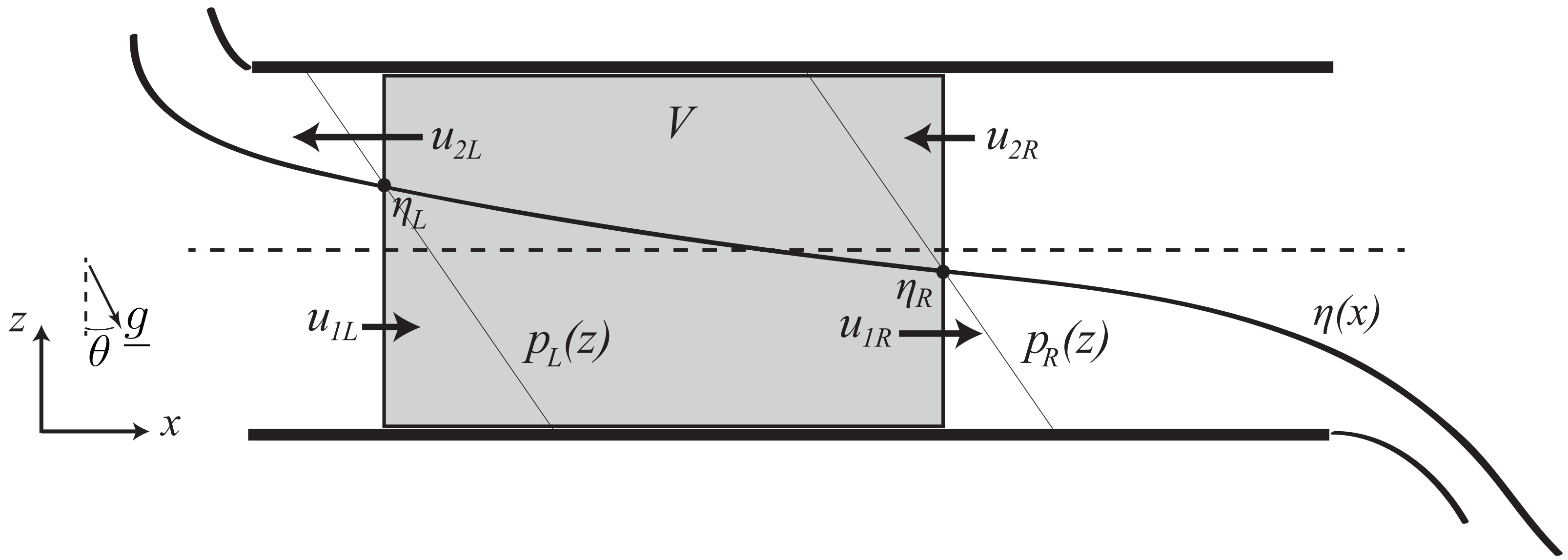
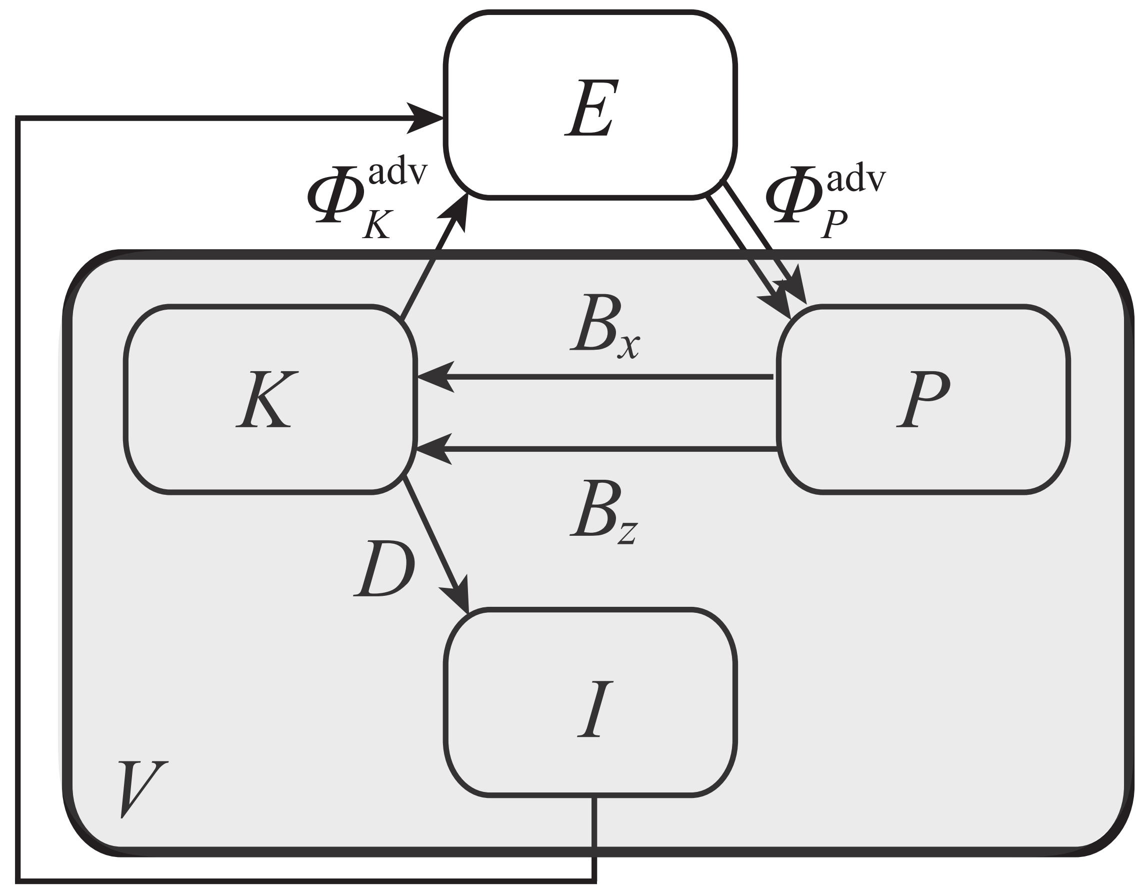
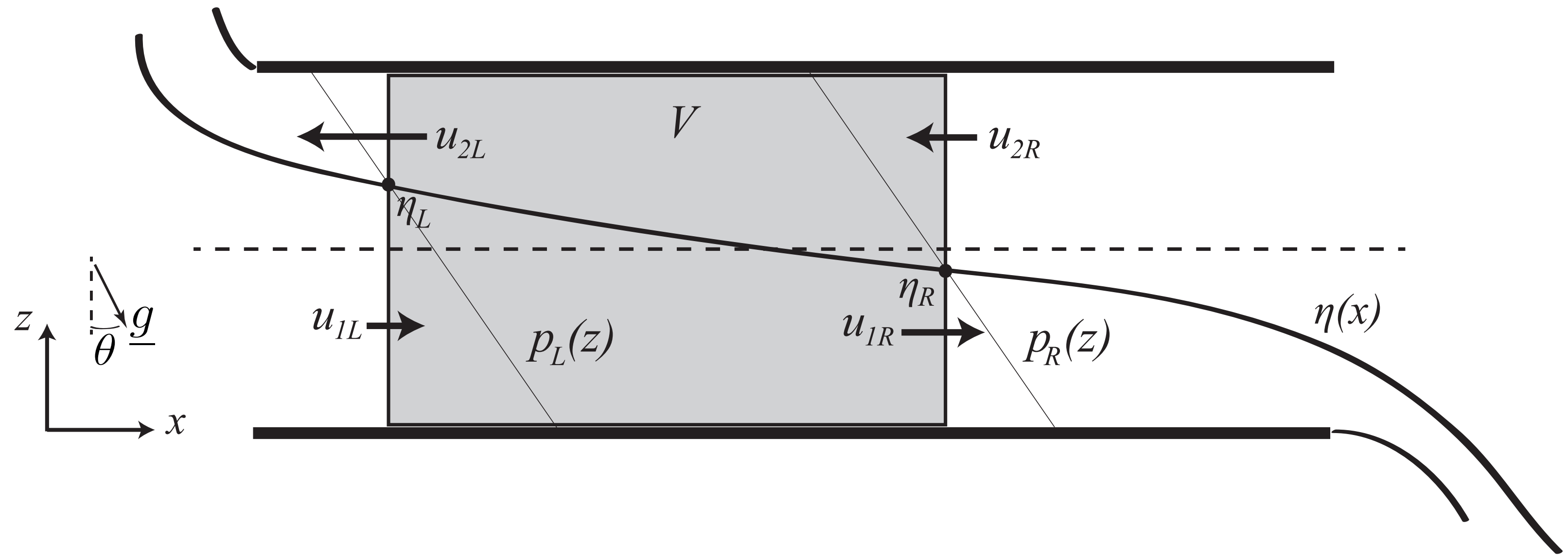


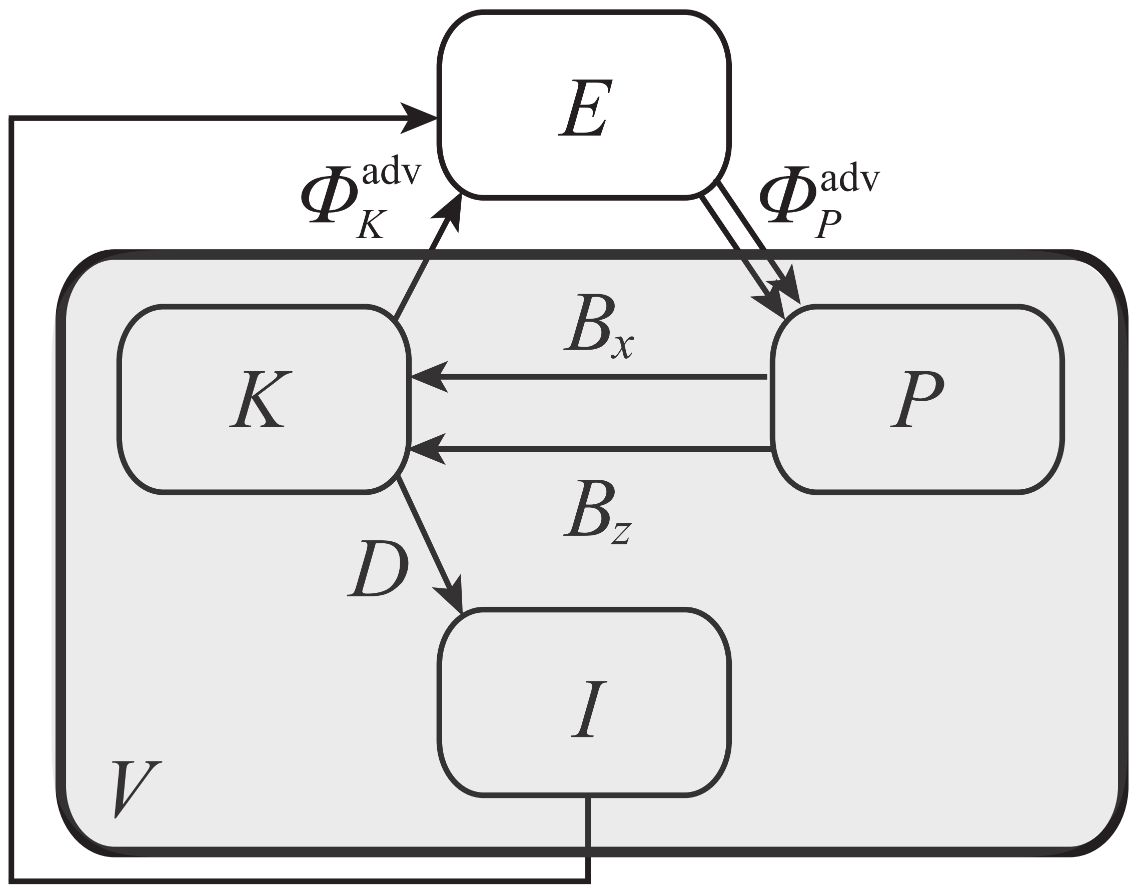


Simplification #2

- Two-layer, near-hydrostatic
- High Re and Pr

- 'Forced' flow $\theta > \alpha$





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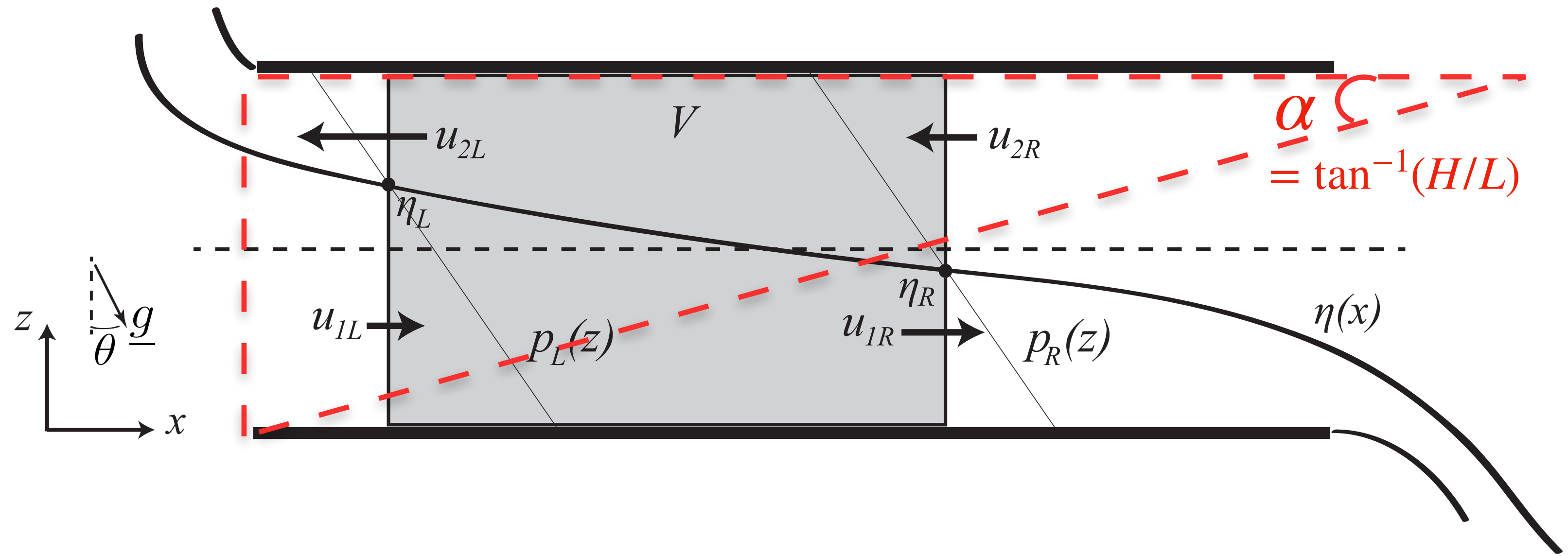
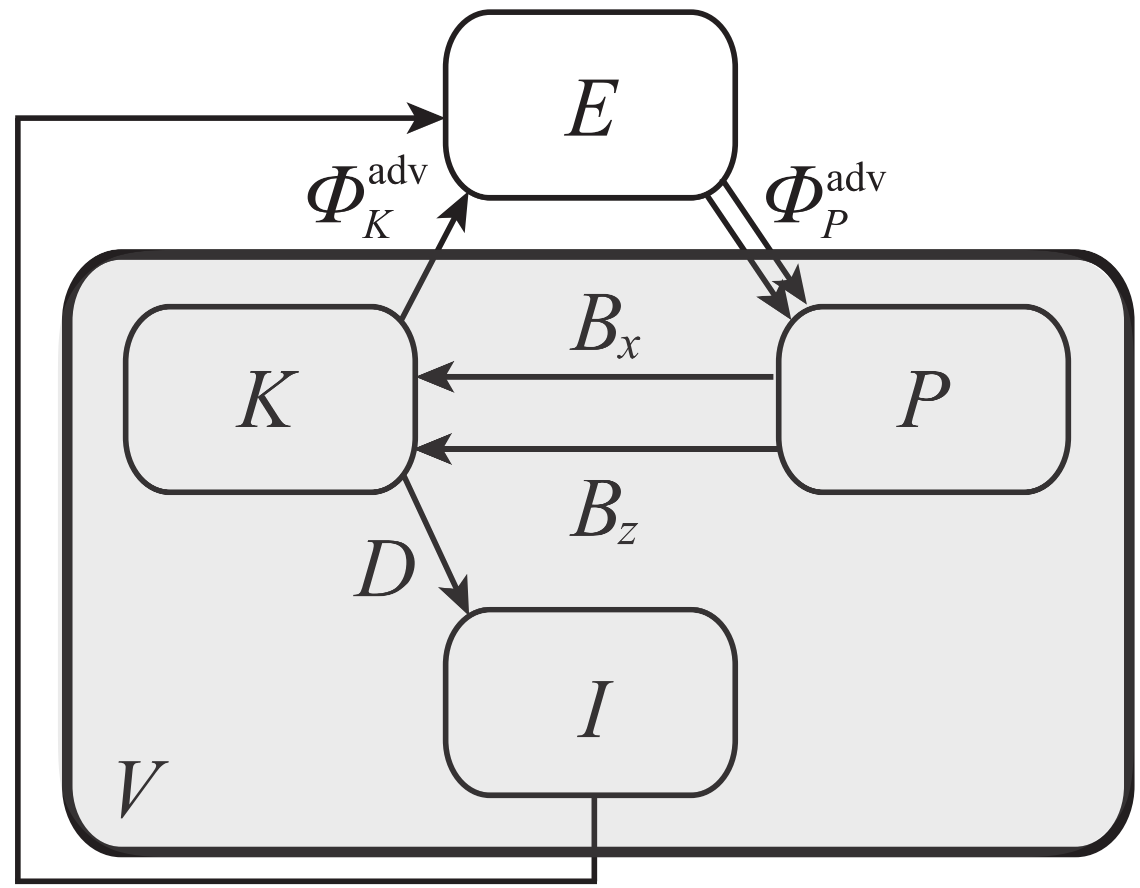
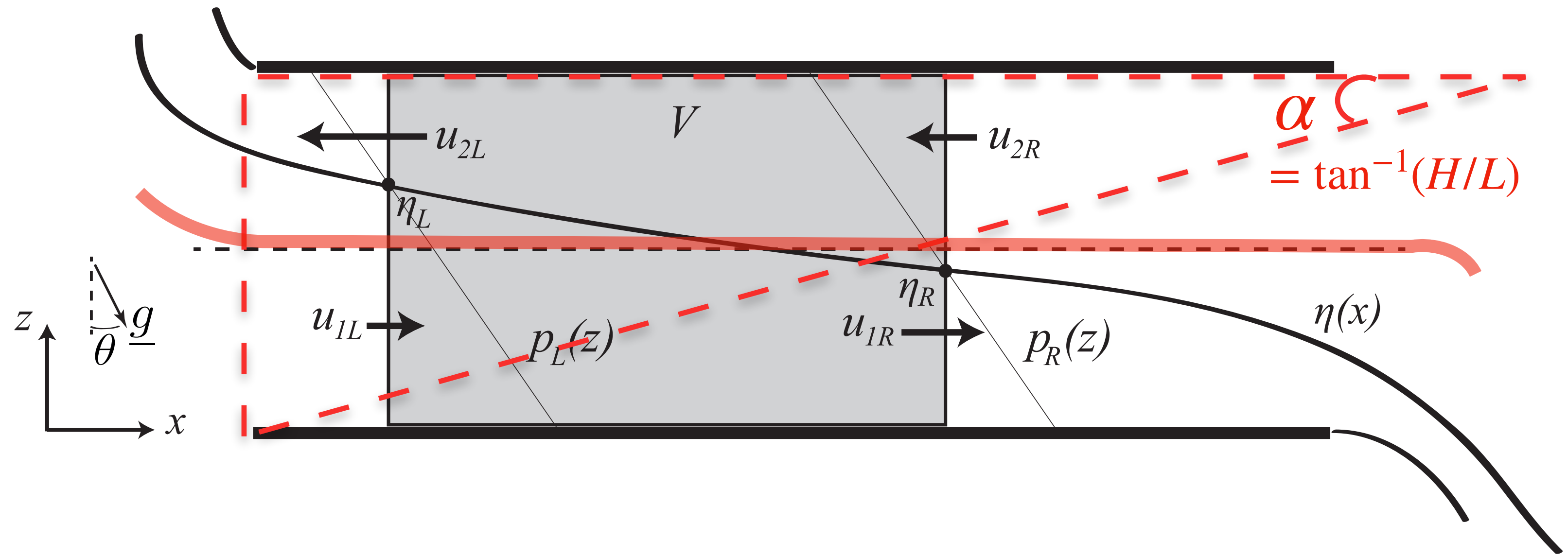


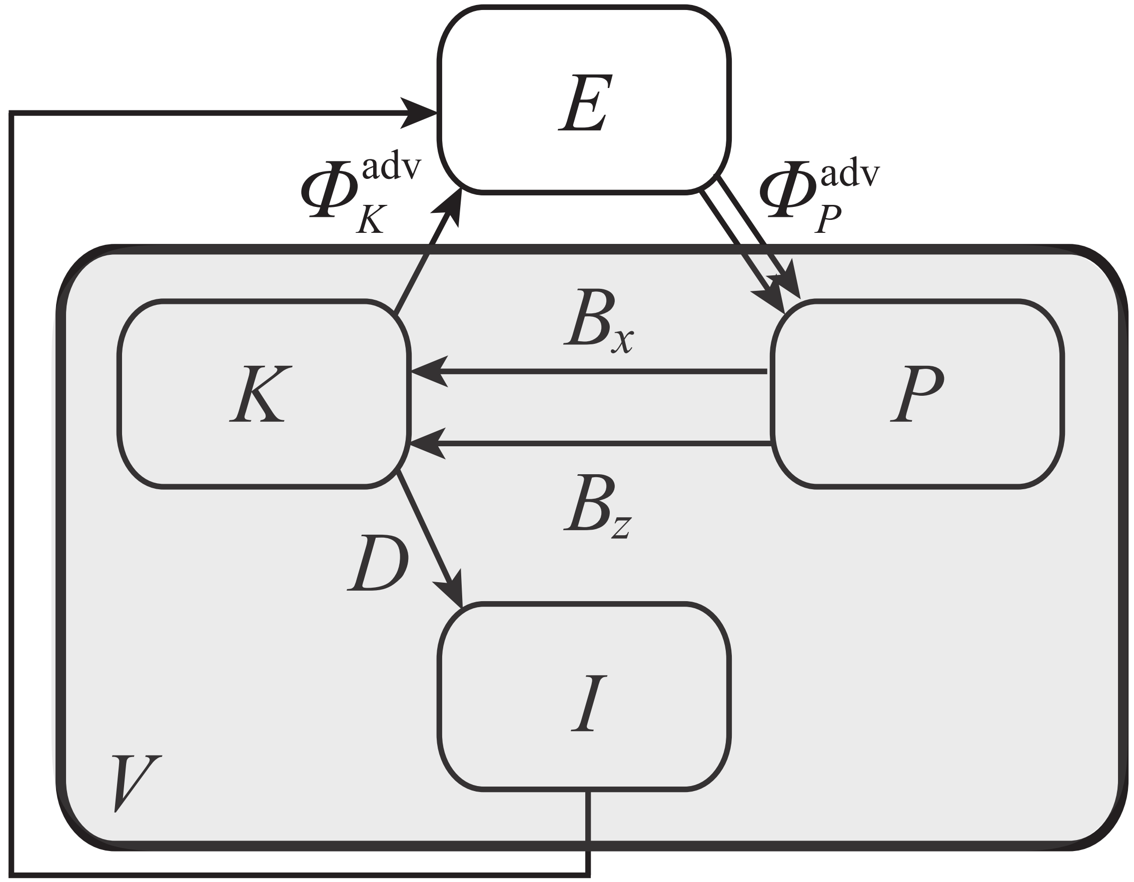


Simplification #2

- Two-layer, near-hydrostatic
- High Re and Pr

- 'Forced' flow $\theta > \alpha$





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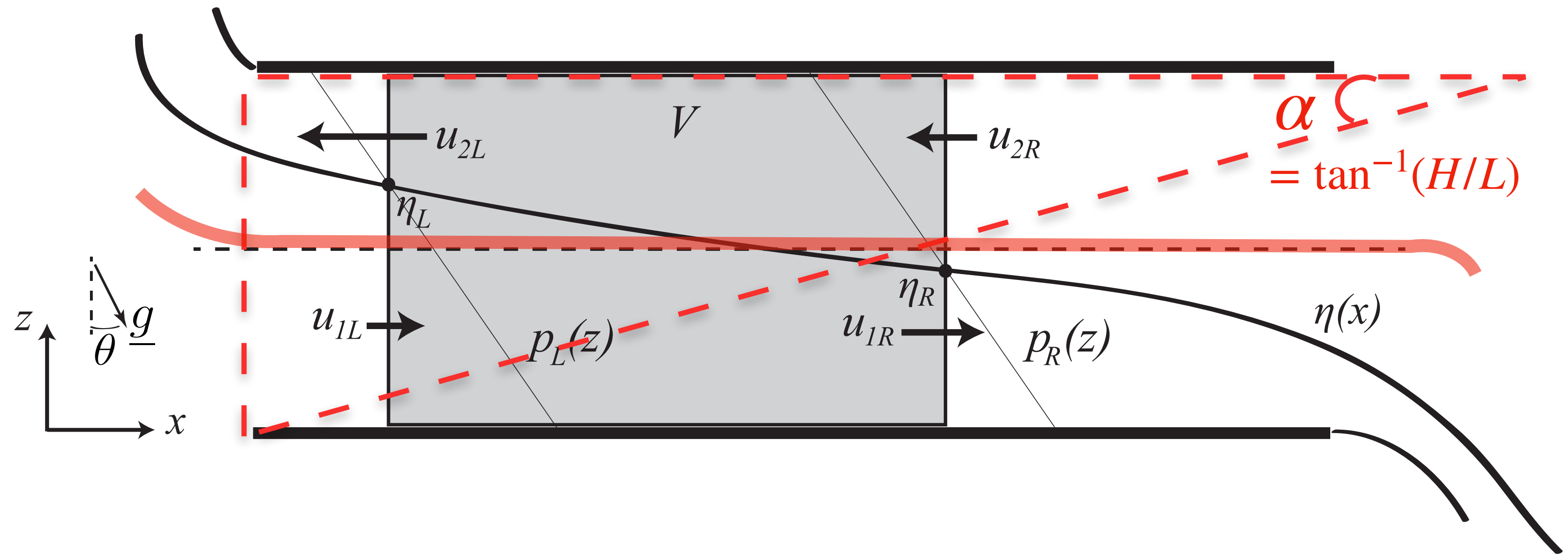

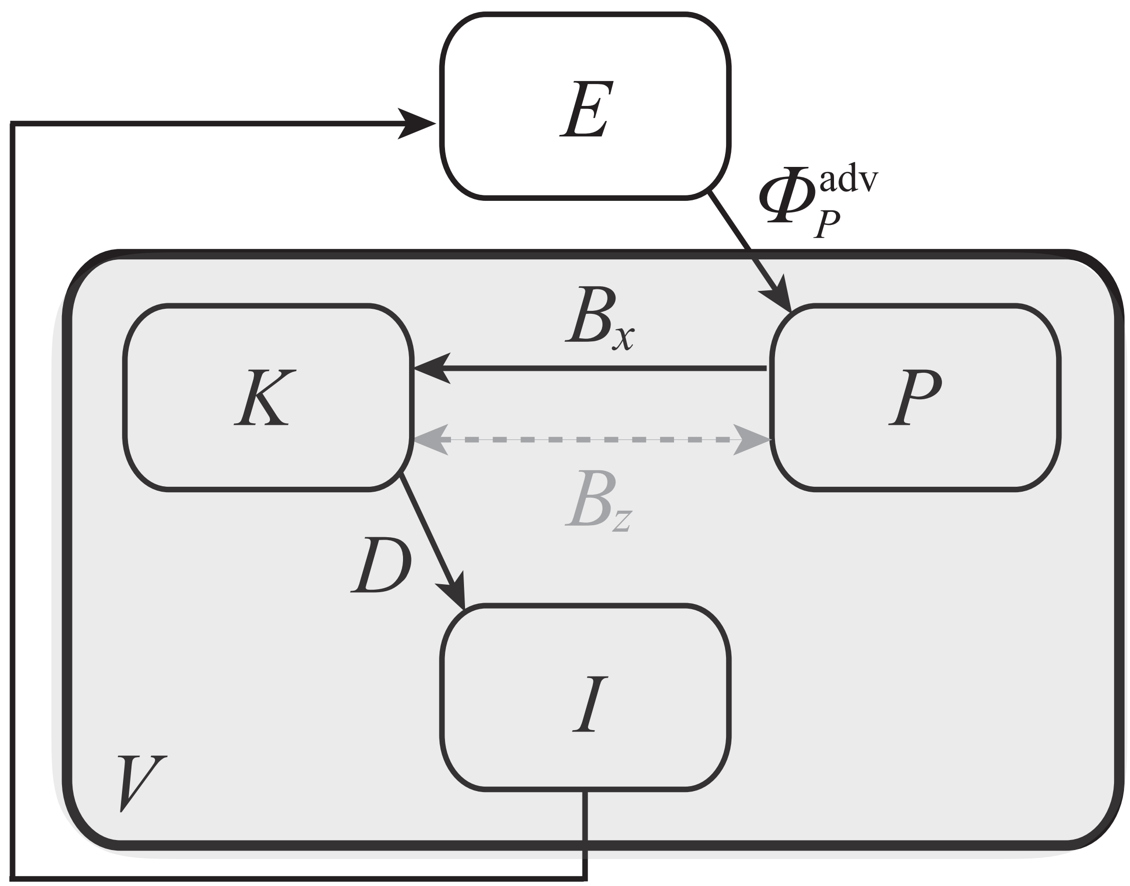

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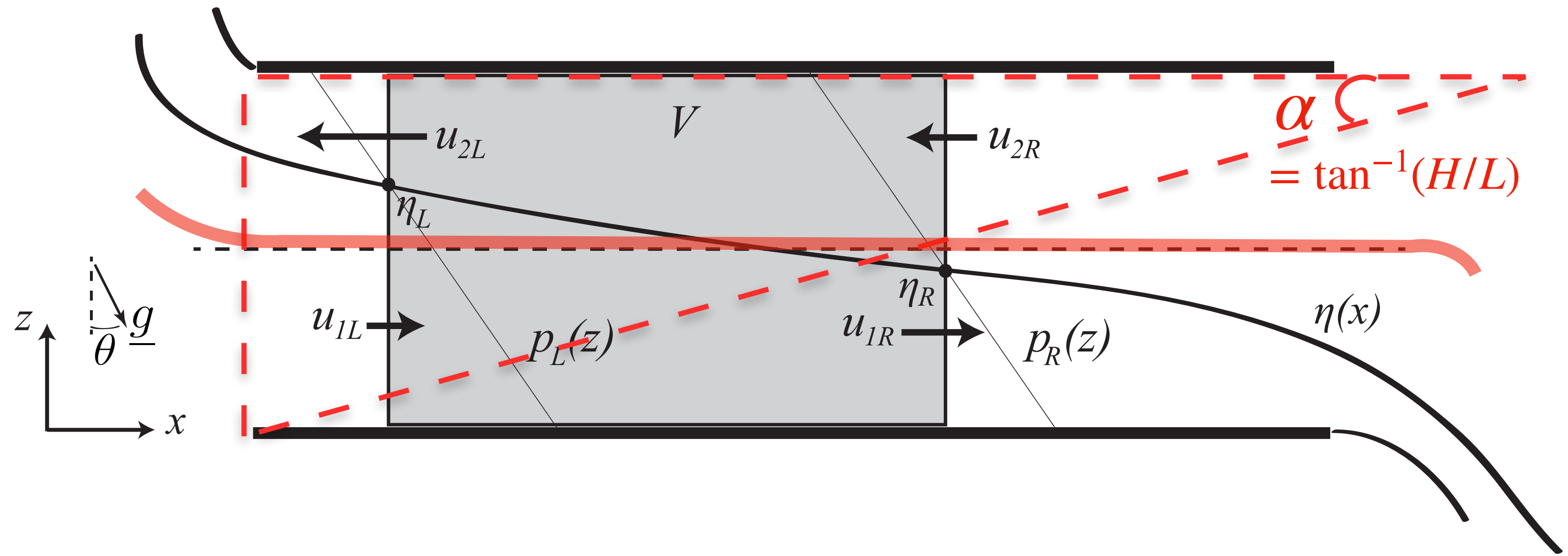
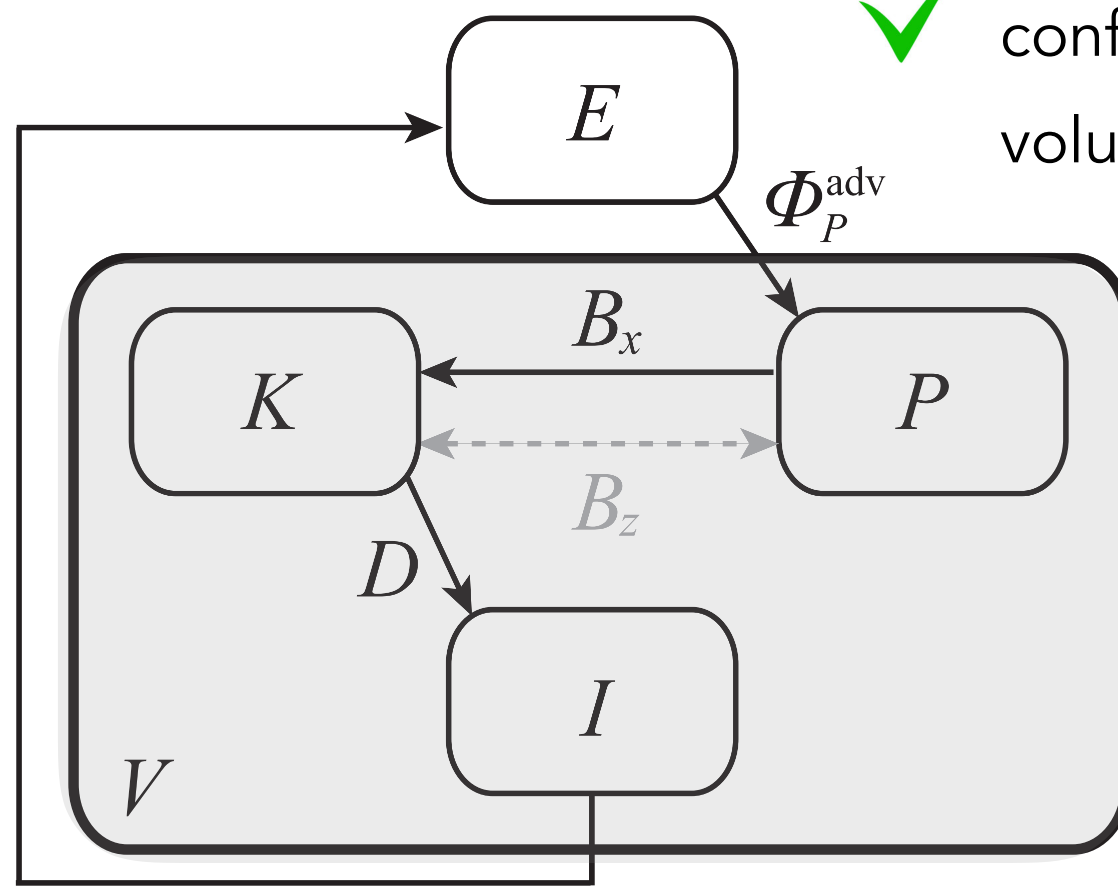


Single power throughput in any duct subvolume

$$\langle D \rangle_t = \frac{1}{4} Q_m \, \theta \approx \frac{1}{8} \theta$$

since the mass flow rate $Q_m \equiv \langle \rho u \rangle_{x,y,z,t} \approx \frac{1}{2}$

due to hydraulic control/Froude condition

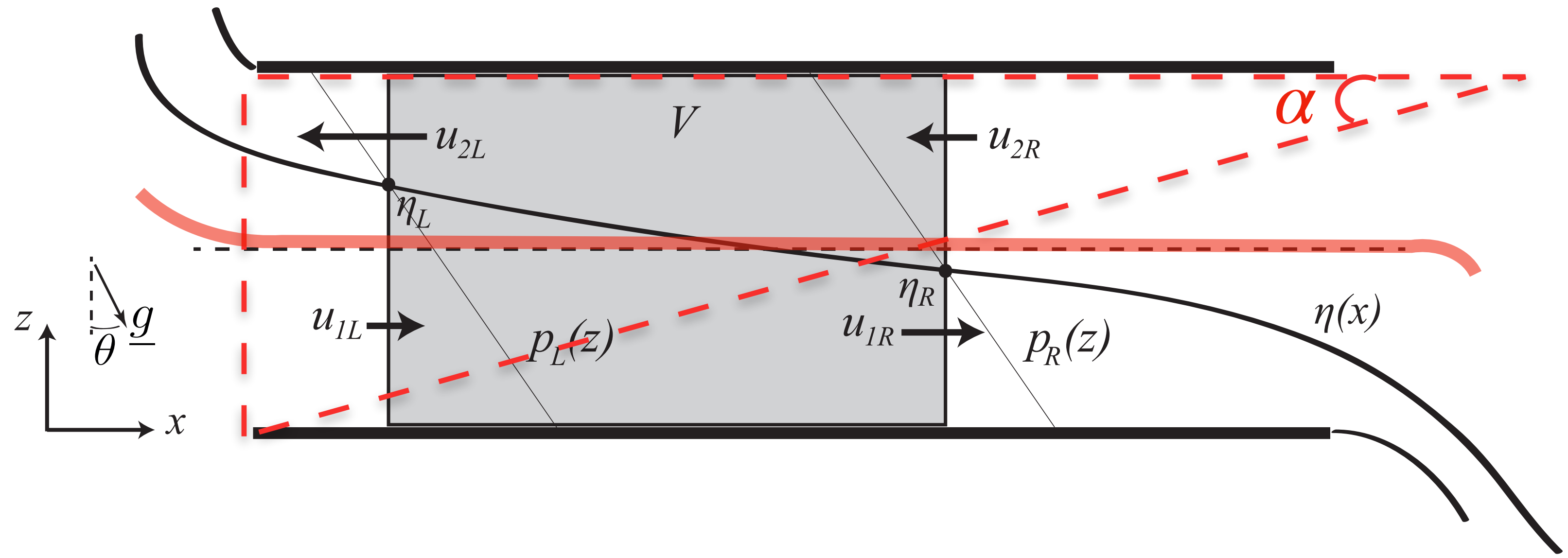


✓ confirmed by volumetric data

2D/3D kinetic energy budgets

- Two-layer, near-hydrostatic
- High Re and Pr

- 'Forced' flow $\theta > \alpha$



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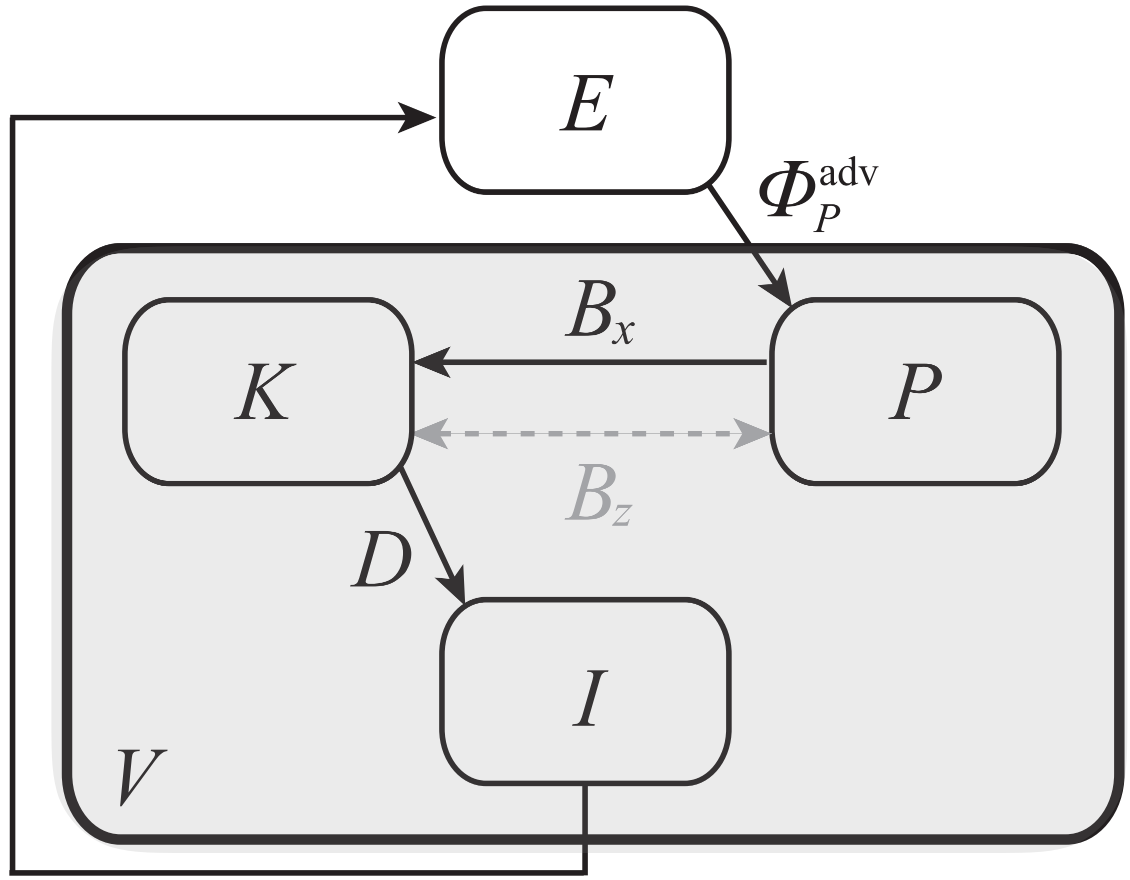

2D/3D kinetic energy budgets

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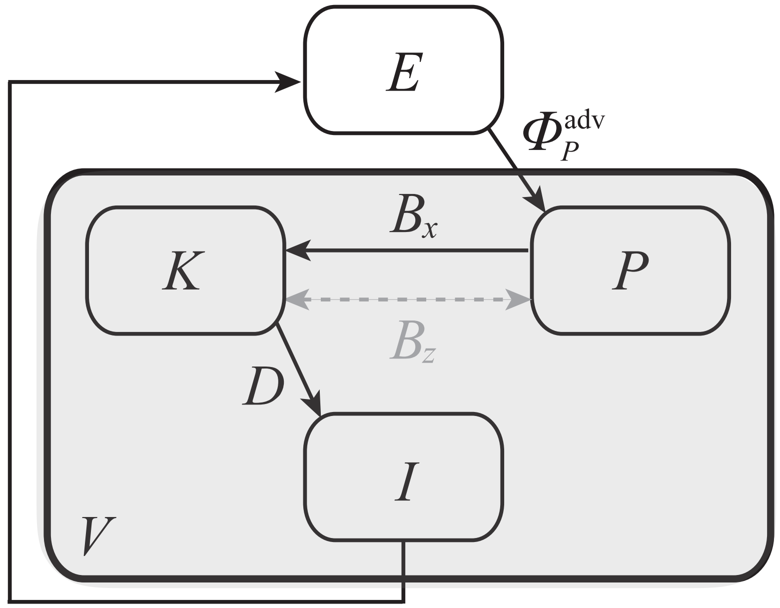
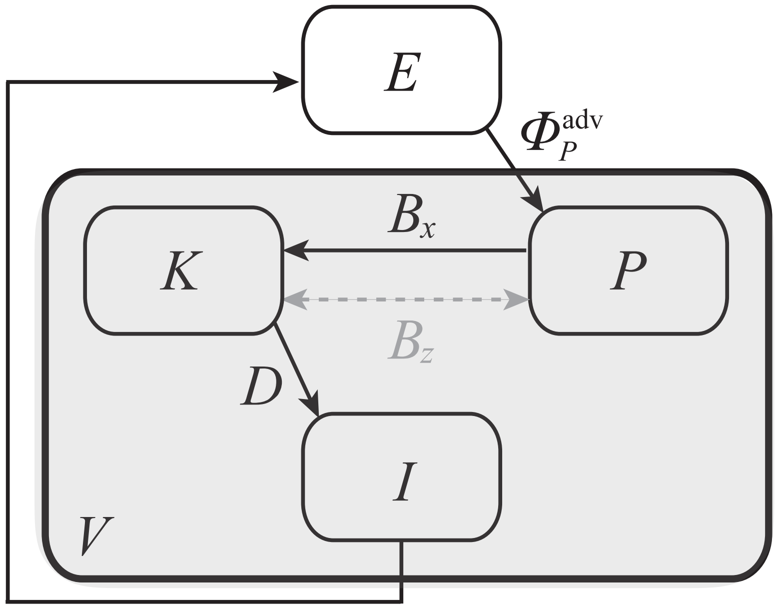


2D/3D kinetic energy budgets

Single power throughput in any duct subvolume

$$\langle D \rangle_t = \frac{1}{4}\, Q_m\, \theta \approx \frac{1}{8}\theta$$

$$\frac{Re}{2}\langle D \rangle_t = \langle s_{ij} s_{ij} \rangle_{x,y,z,t} \approx \frac{1}{16}\theta\, Re$$



2D/3D kinetic energy budgets

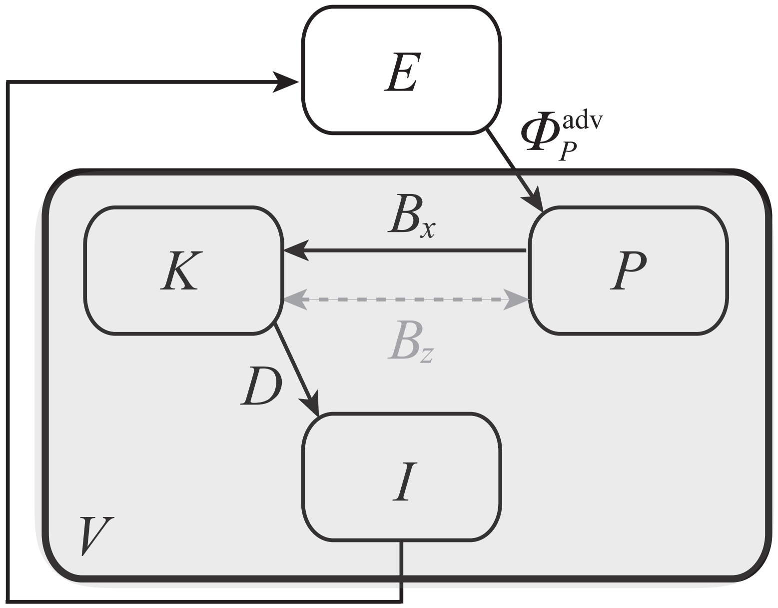
Single power throughput in any duct subvolume

$$\langle D \rangle_t = \frac{1}{4}\, Q_m\, \theta \approx \frac{1}{8}\theta$$

$$\frac{Re}{2}\langle D \rangle_t = \langle \mathsf{s}_{ij}\mathsf{s}_{ij} \rangle_{x,y,z,t} \approx \frac{1}{16}\theta\, Re$$

$$\langle \mathsf{s}_{ij}^{2d}\mathsf{s}_{ij}^{2d} \rangle_{y,z,t} + \langle \mathsf{s}_{ij}^{3d}\mathsf{s}_{ij}^{3d} \rangle_{x,y,z,t} \approx \frac{1}{16}\theta\, Re$$

('2D' = x-averaged)



Single power throughput in any duct subvolume

$$\langle D\rangle_t = \frac{1}{4}\,Q_m\,\theta \approx \frac{1}{8}\theta$$

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$$\langle \mathsf{s}^{2d}_{ij}\mathsf{s}^{2d}_{ij}\rangle_{y,z,t} + \langle \mathsf{s}^{3d}_{ij}\mathsf{s}^{3d}_{ij}\rangle_{x,y,z,t} \approx \frac{1}{16}\theta\,Re$$

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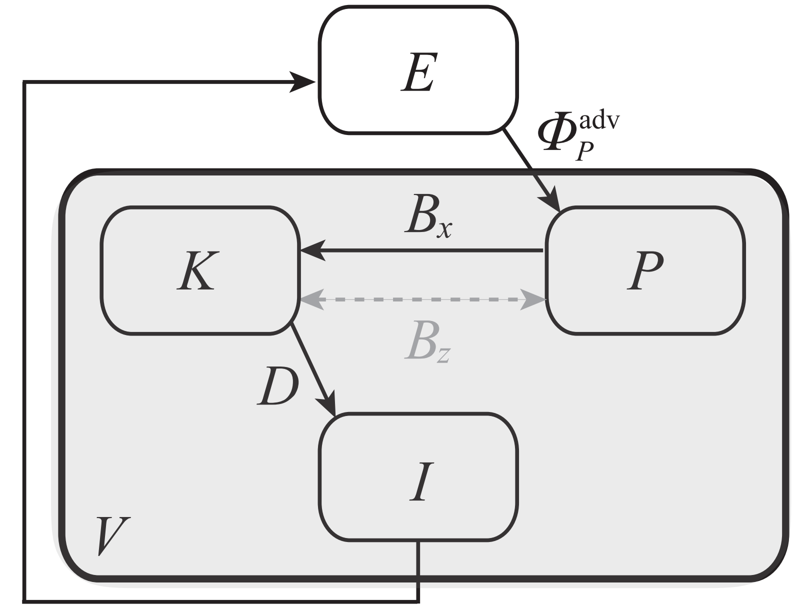
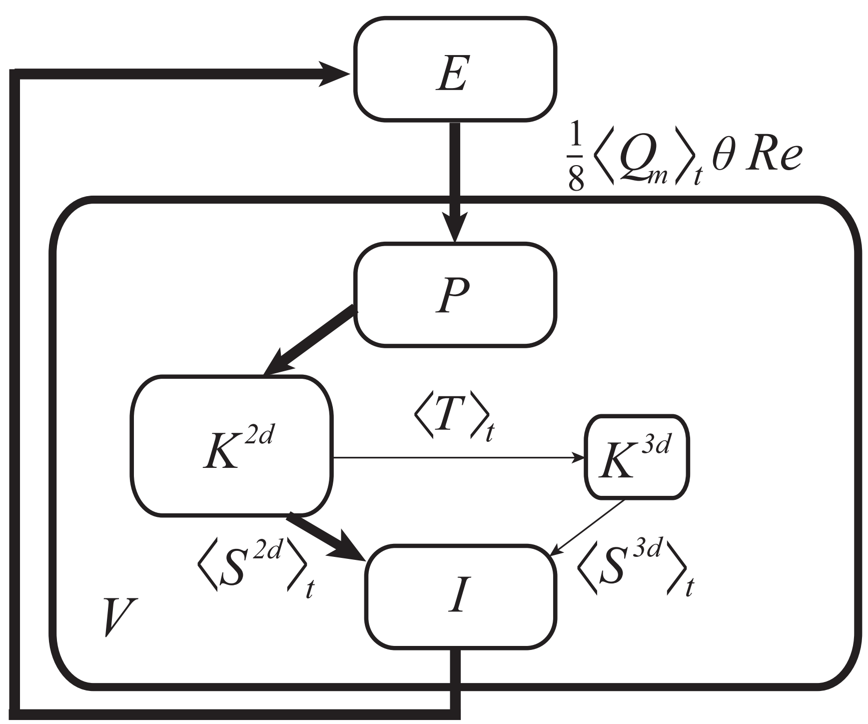
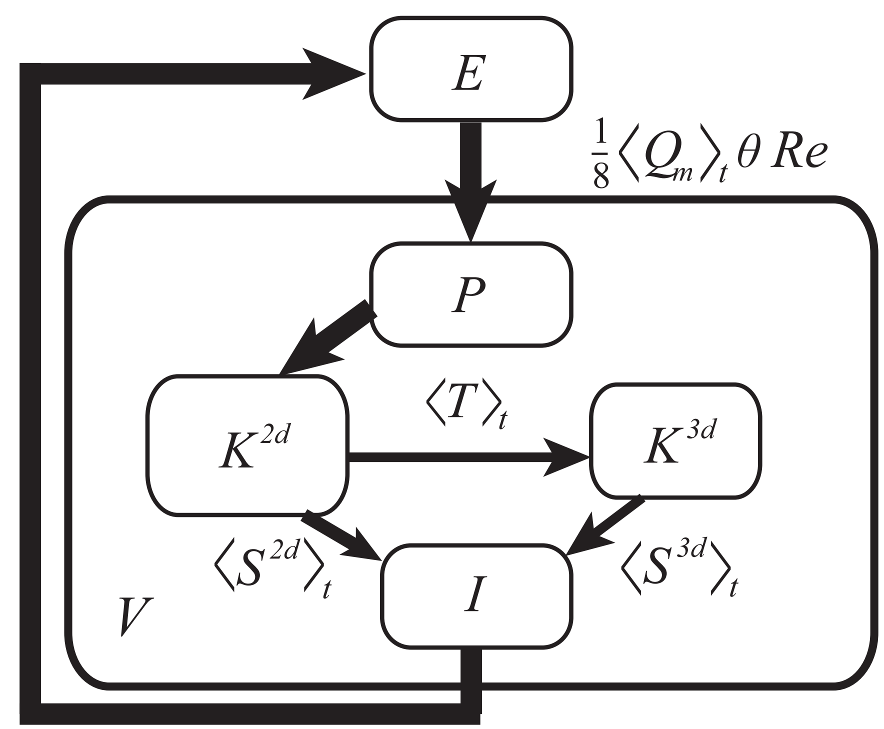


Low $\theta\,Re$



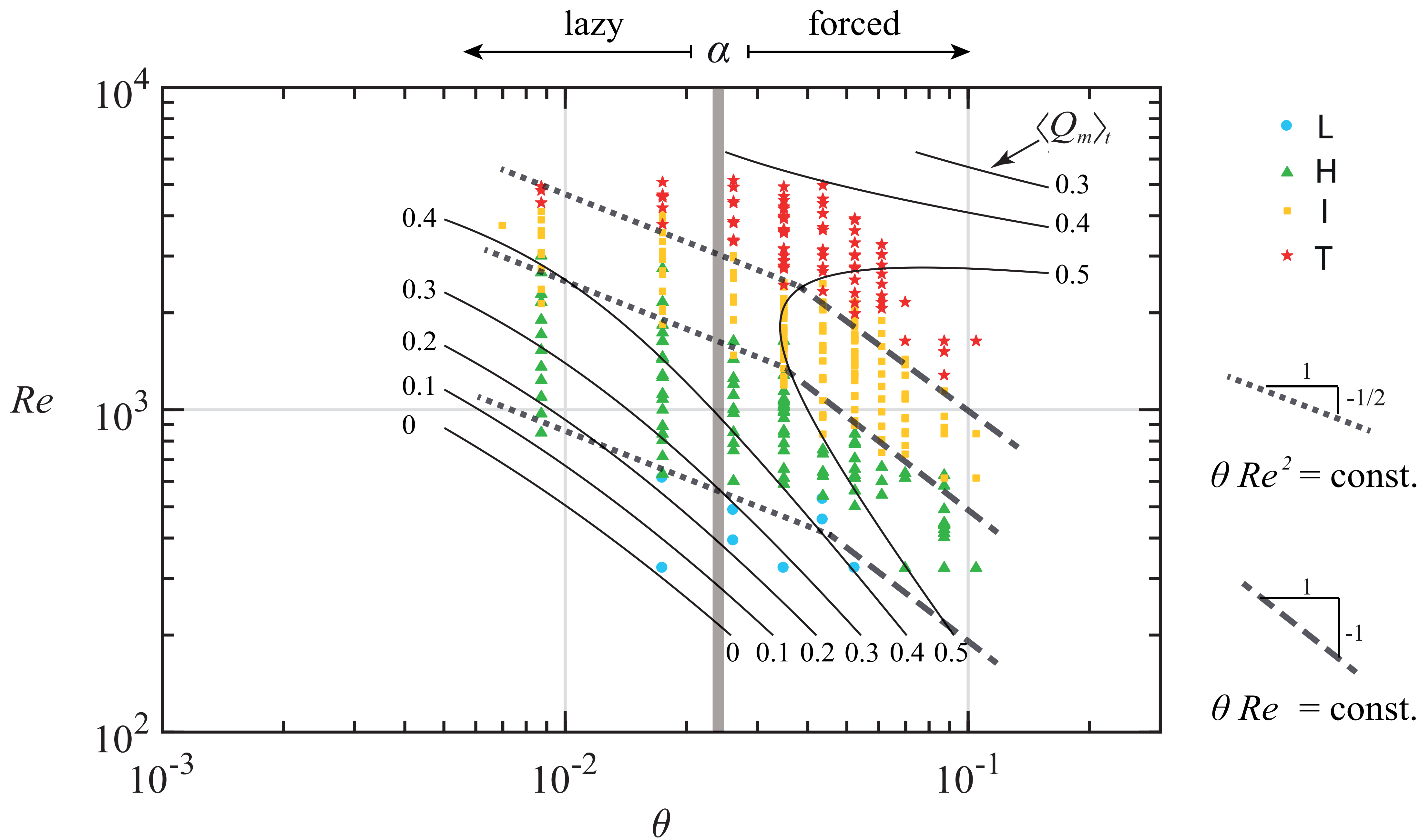
High $\theta\,Re$



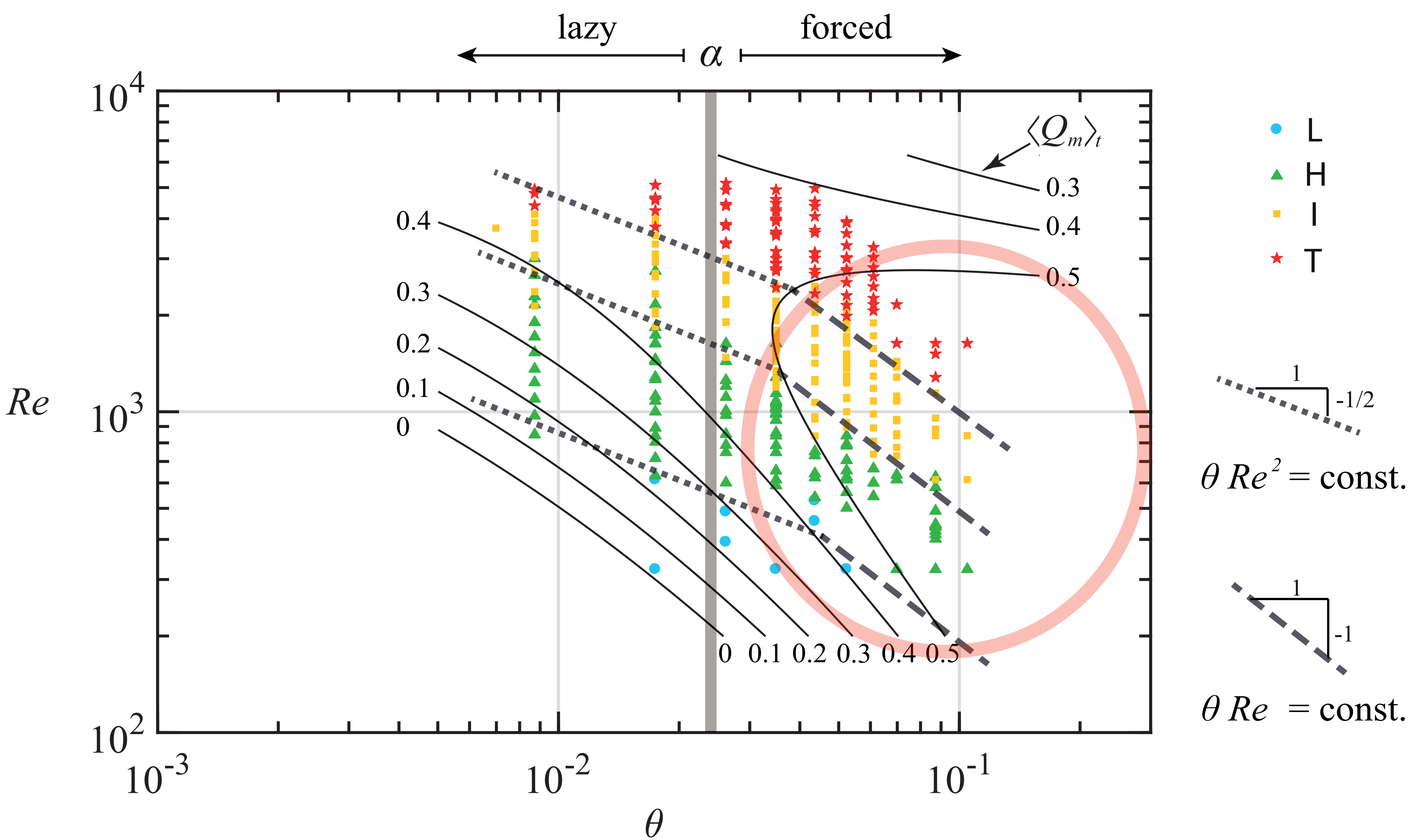
Hypothesis: regime transitions are **caused by** S^{3d} following a plateau in S^{2d} and **scale with** $\theta\,Re$

Experimental validation of the $\theta\, Re$ scaling

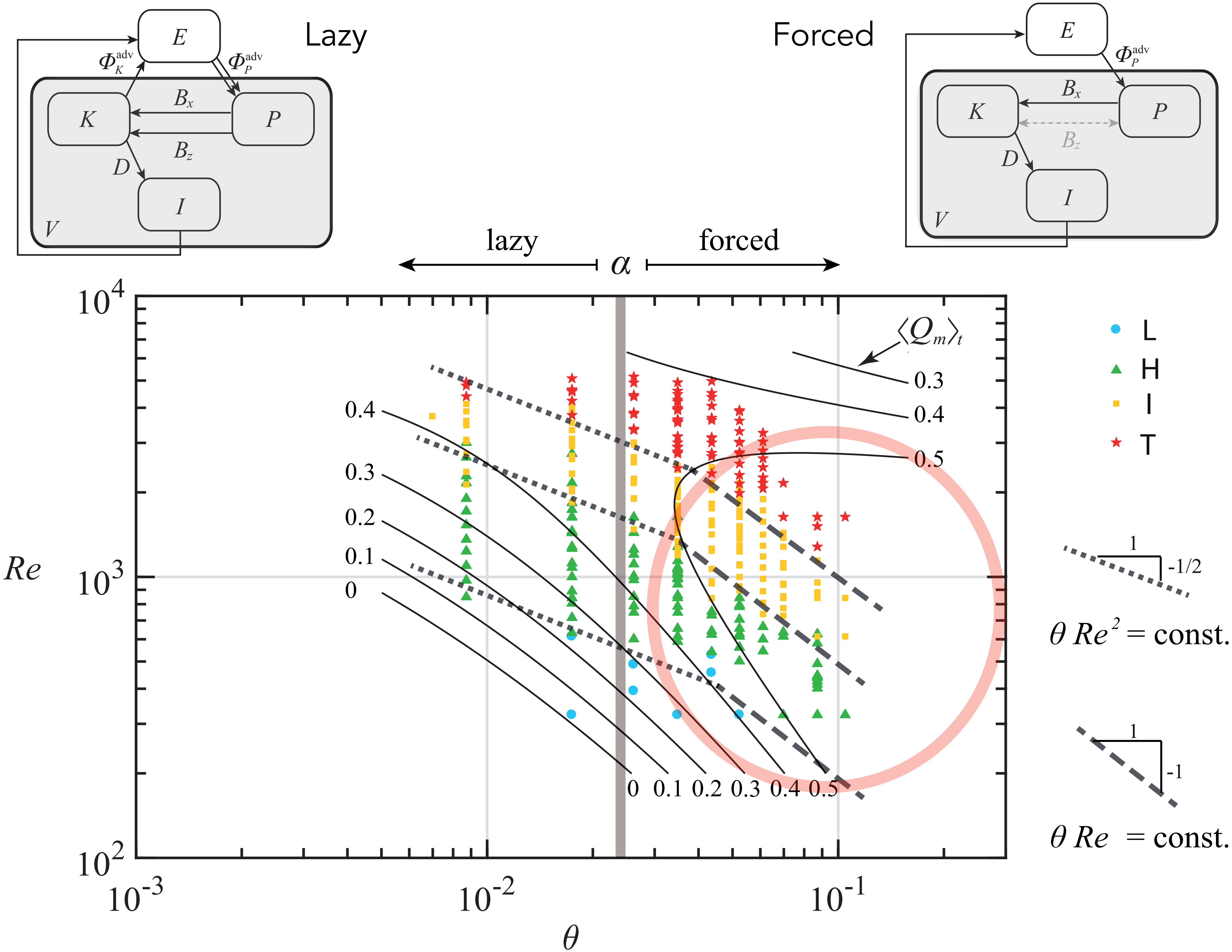
Experimental validation of the $\theta\, Re$ scaling



Experimental validation of the $\theta\, Re$ scaling

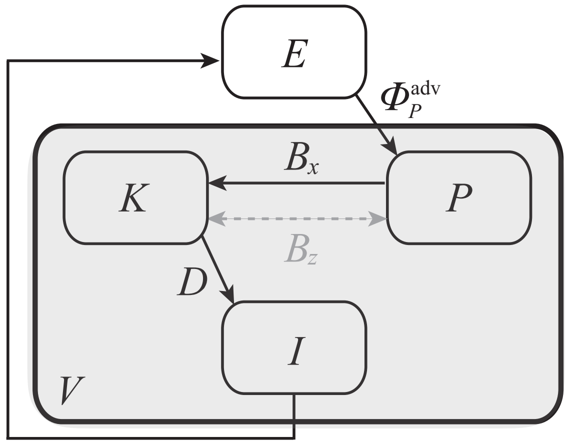


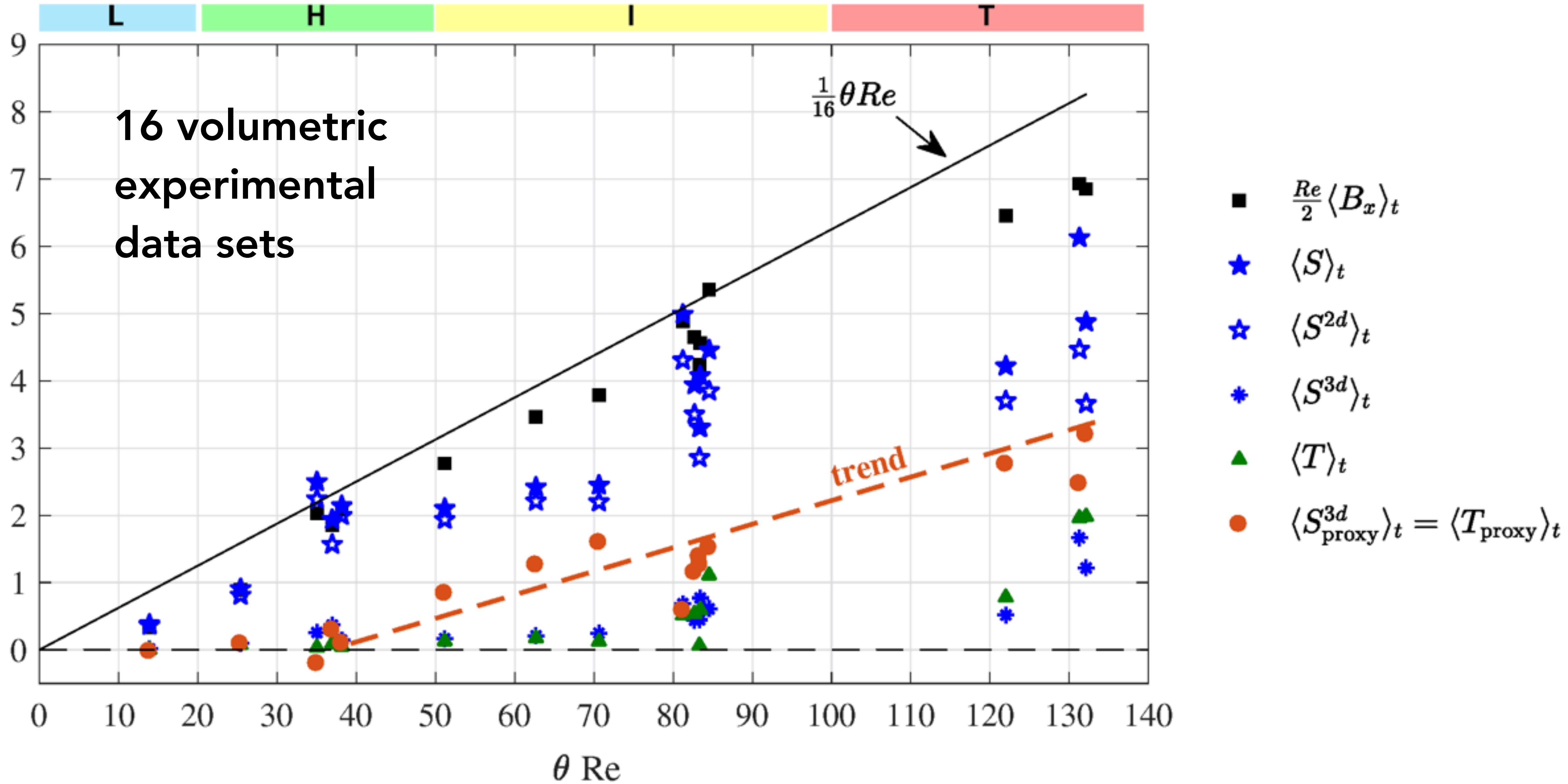
Experimental validation of the $\theta\, Re$ scaling



Experimental validation of the model and hypothesis

Forced





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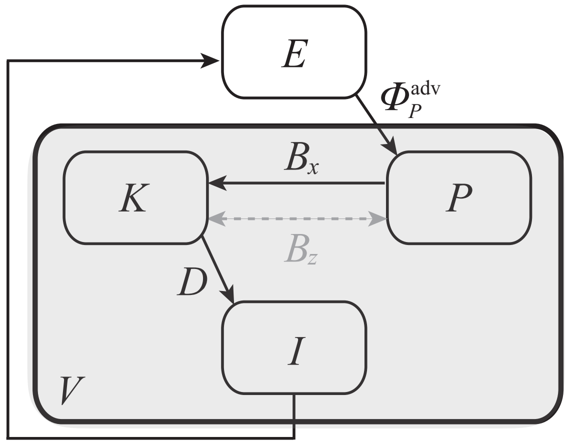
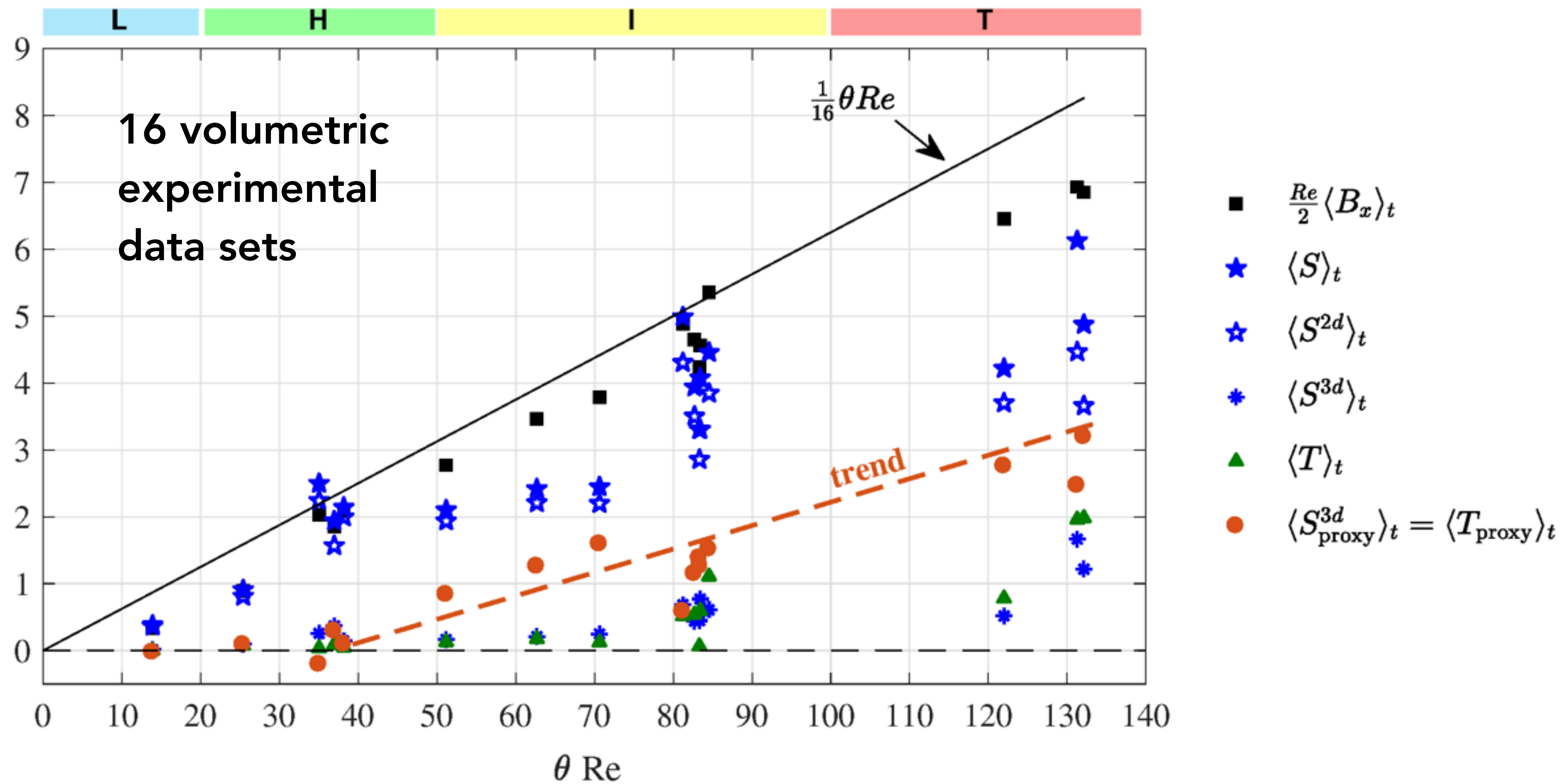


Further details about the transitions:

- L → H: Holmboe waves caused by increase in S^{2d}
- H → I and I → T: caused by increase in S^{3d}



Experiment visualisation of energy dissipation

Holmboe regime

$\theta = 1^{\circ}, \ Re = 1455$

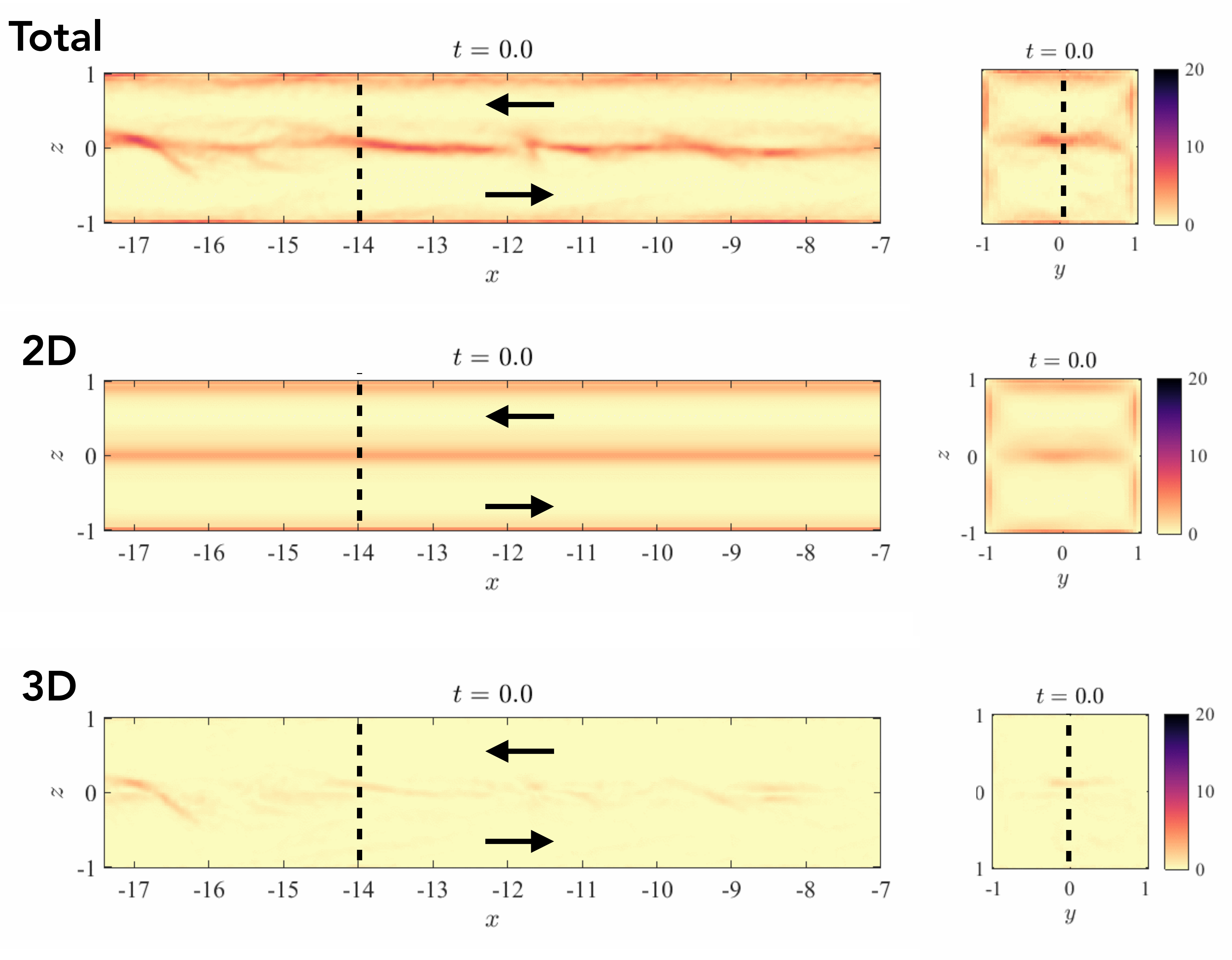
$\mathsf{s}_{ij}\mathsf{s}_{ij}(x, y, z, t)$

$=$

$\mathsf{s}^{2d}_{ij}\mathsf{s}^{2d}_{ij}(y, z, t)$

$+$

$\mathsf{s}^{3d}_{ij}\mathsf{s}^{3d}_{ij}(x, y, z, t)$



Experiment visualisation of energy dissipation

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$\theta = 1^\circ, \quad Re = 1455$

$s_{ij}s_{ij}(x, y, z, t)$

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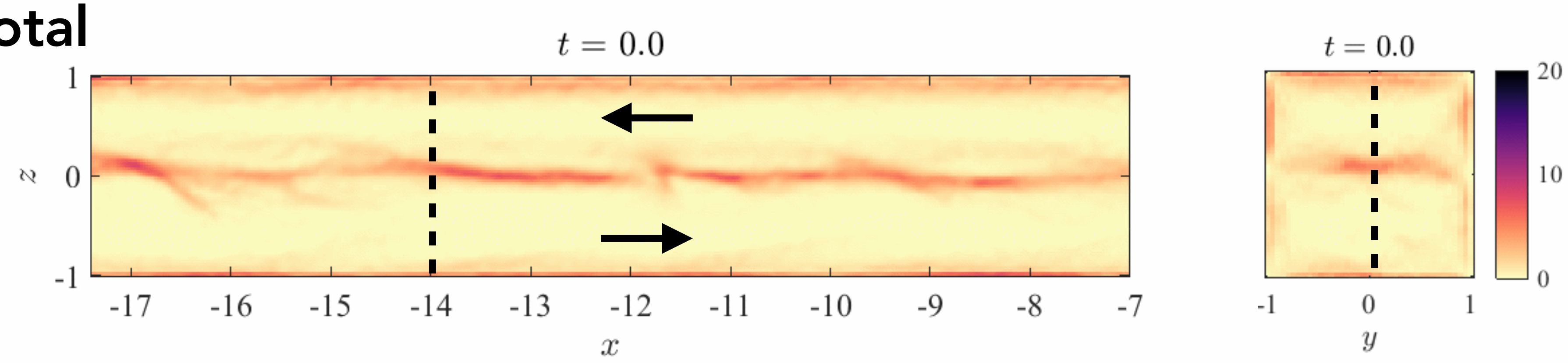
$s_{ij}^{2d}s_{ij}^{2d}(y, z, t)$

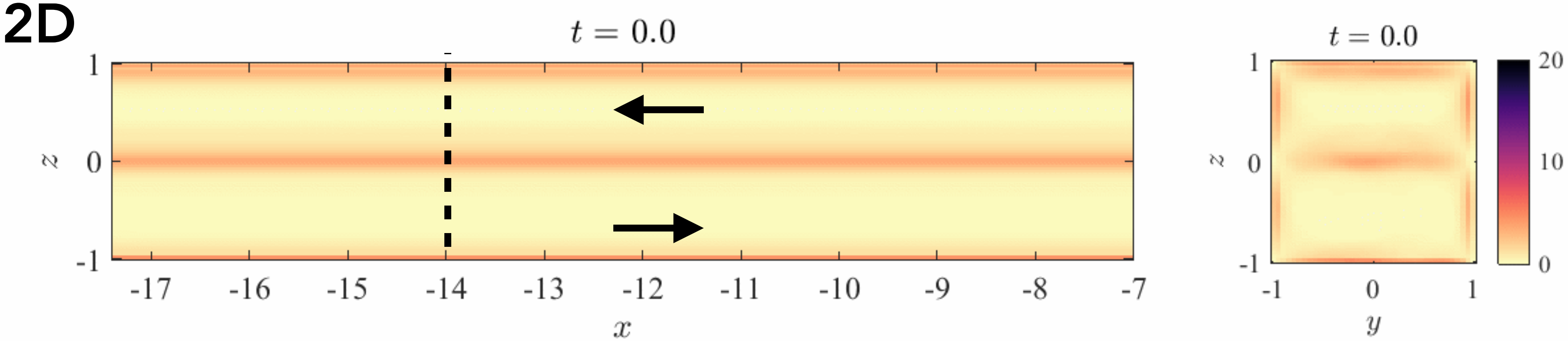
$+$

$s_{ij}^{3d}s_{ij}^{3d}(x, y, z, t)$

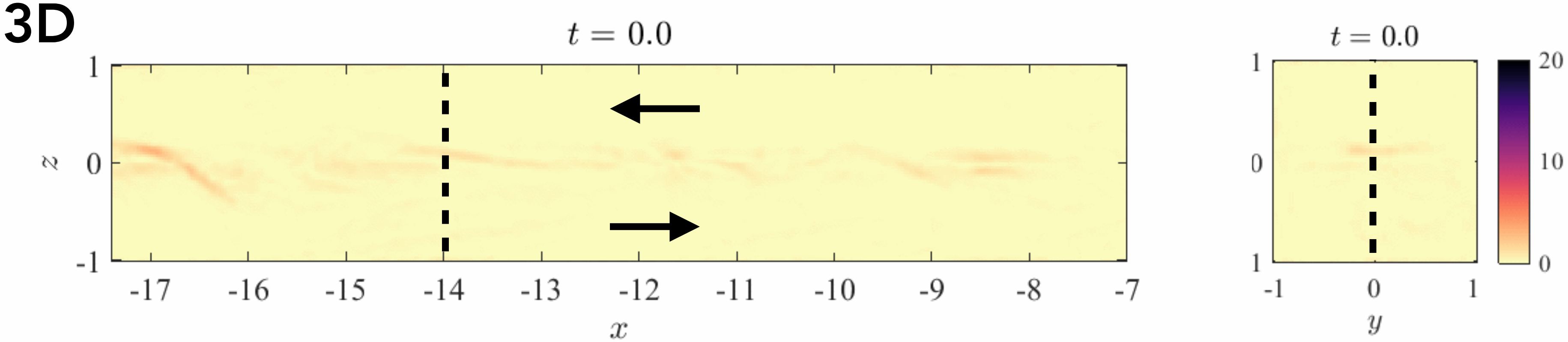
Total



2D



3D



Turbulent regime

$\theta = 6^\circ, \quad Re = 1256$

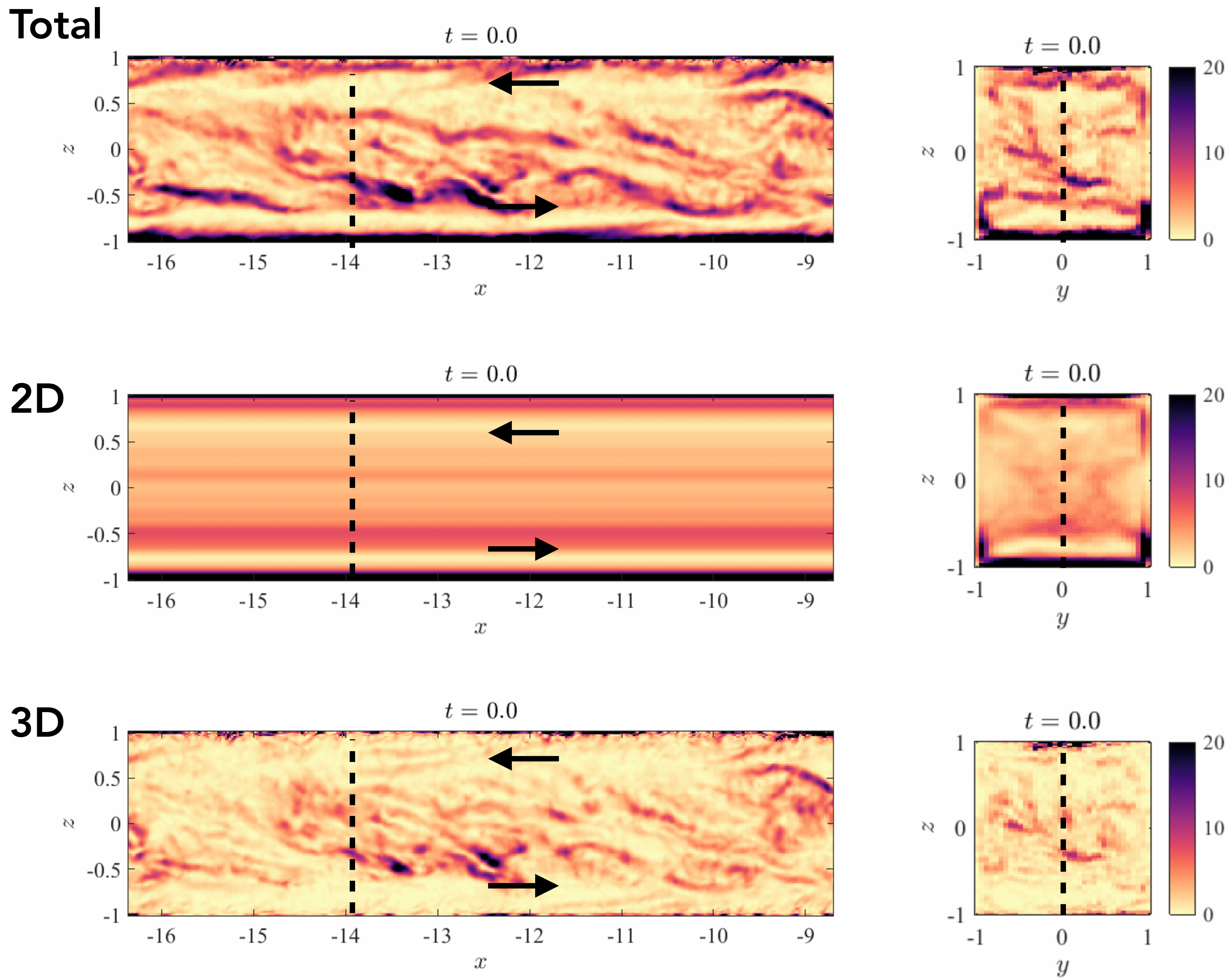
$$\mathsf{s}_{ij}\mathsf{s}_{ij}(x, y, z, t)$$

$$=$$

$$\mathsf{s}^{2d}_{ij}\mathsf{s}^{2d}_{ij}(y, z, t)$$

$$+$$

$$\mathsf{s}^{3d}_{ij}\mathsf{s}^{3d}_{ij}(x, y, z, t)$$



Experiment visualisation of energy dissipation

Turbulent regime

$\theta = 6^\circ, \quad Re = 1256$

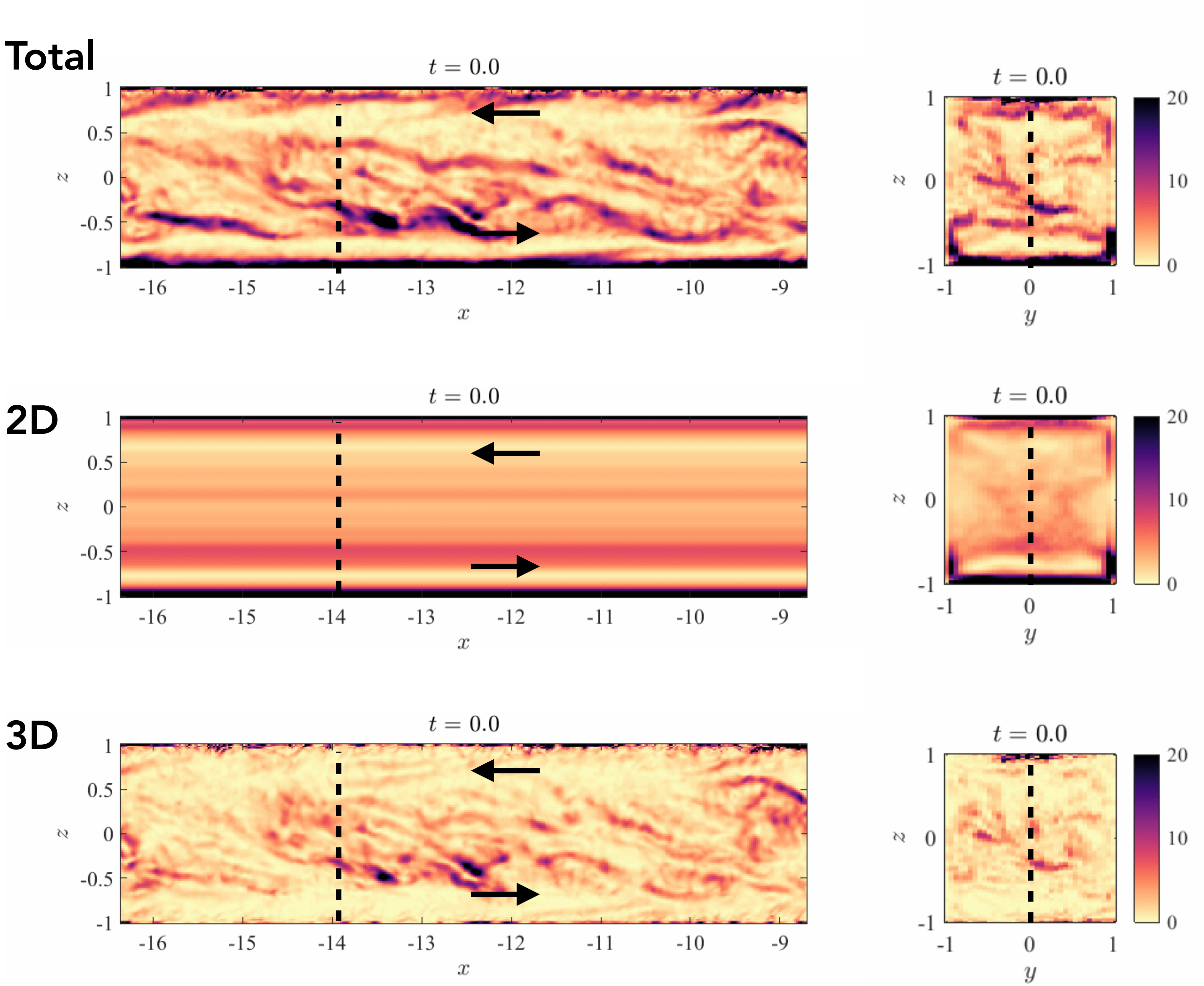
$$s_{ij}s_{ij}(x, y, z, t)$$

$$=$$

$$s_{ij}^{2d}s_{ij}^{2d}(y, z, t)$$

$$+$$

$$s_{ij}^{3d}s_{ij}^{3d}(x, y, z, t)$$



More details in: Lefauve, Partridge & Linden, *J. Fluid Mech.* **875** : 657-698 (2019)