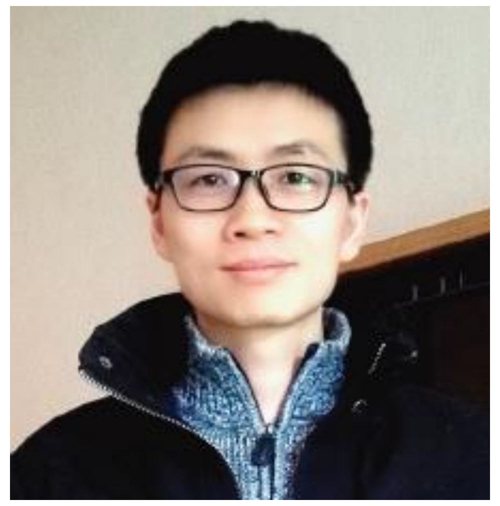
New insights into experimental stratified flows obtained through a physics-informed neural network

Sharing is encouraged

Lu Zhu, Xianyang Jiang, Adrien Lefauve, R. R. Kerswell, P. F. Linden

European Research Council



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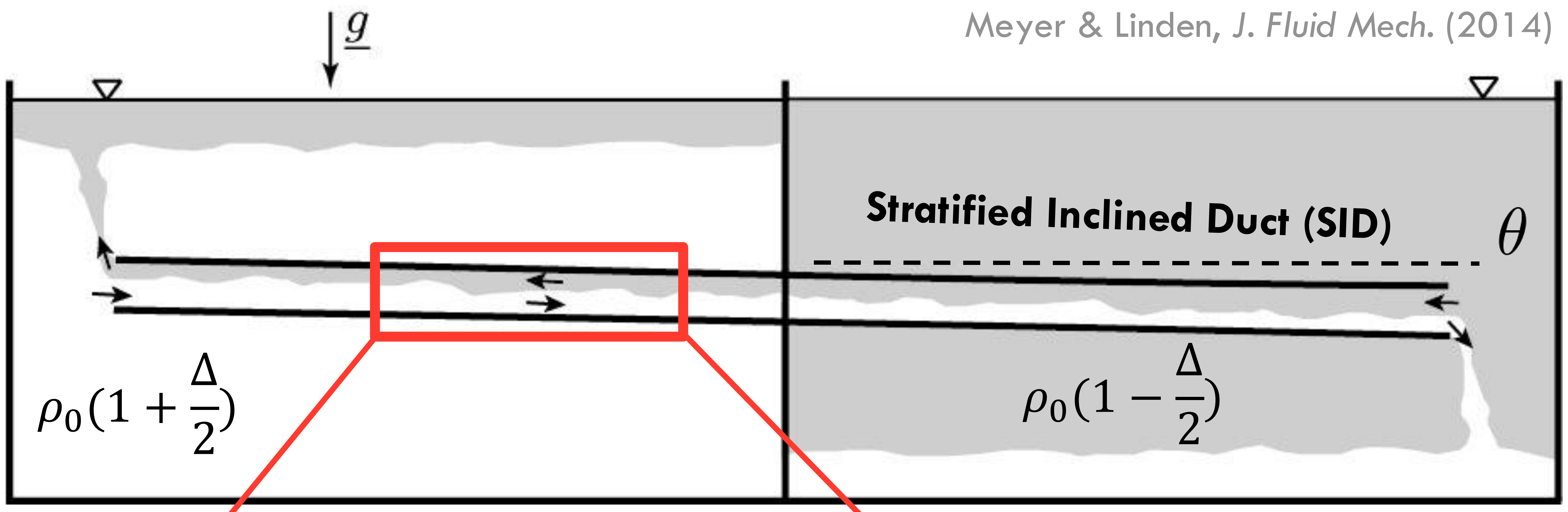
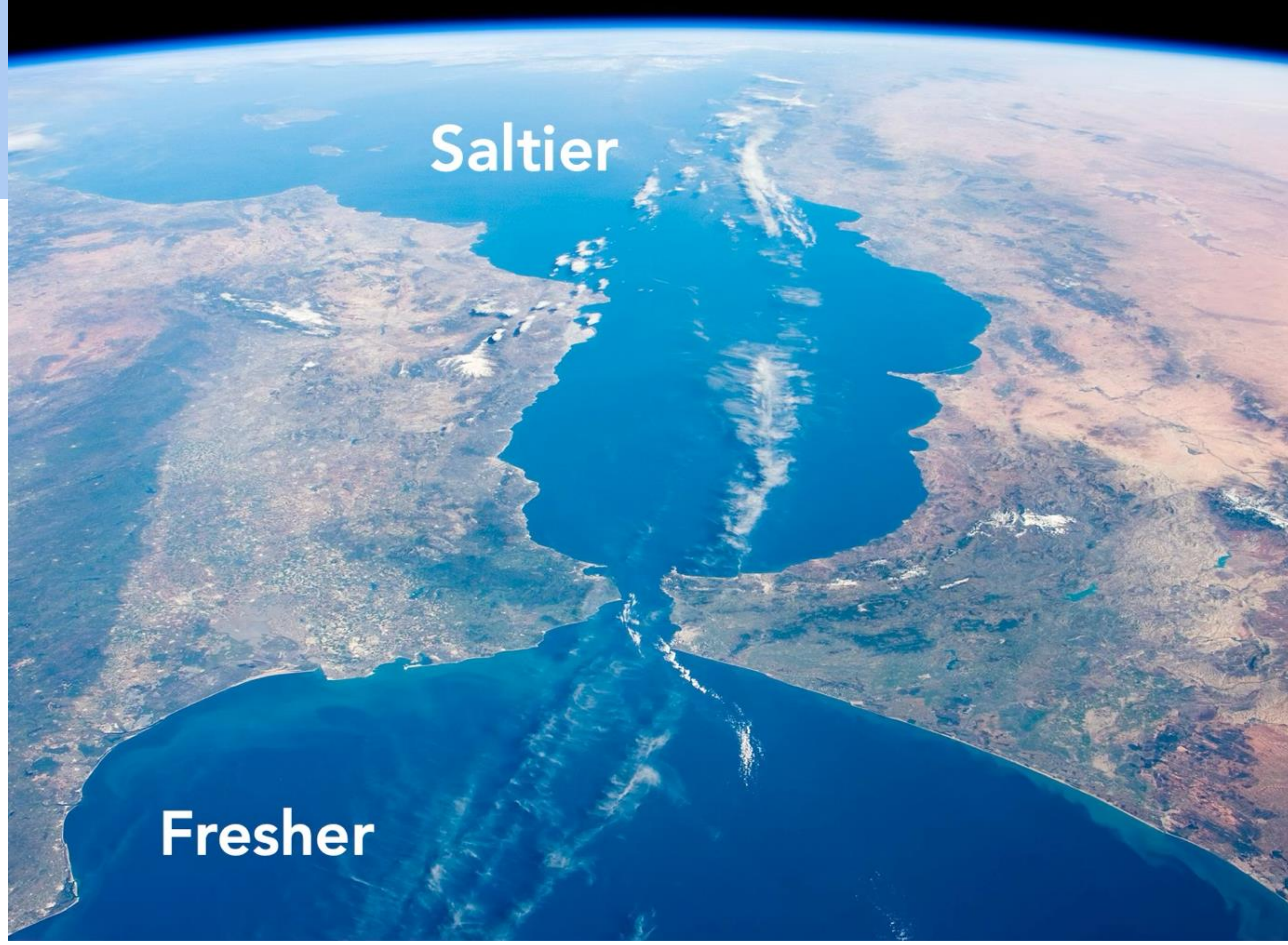
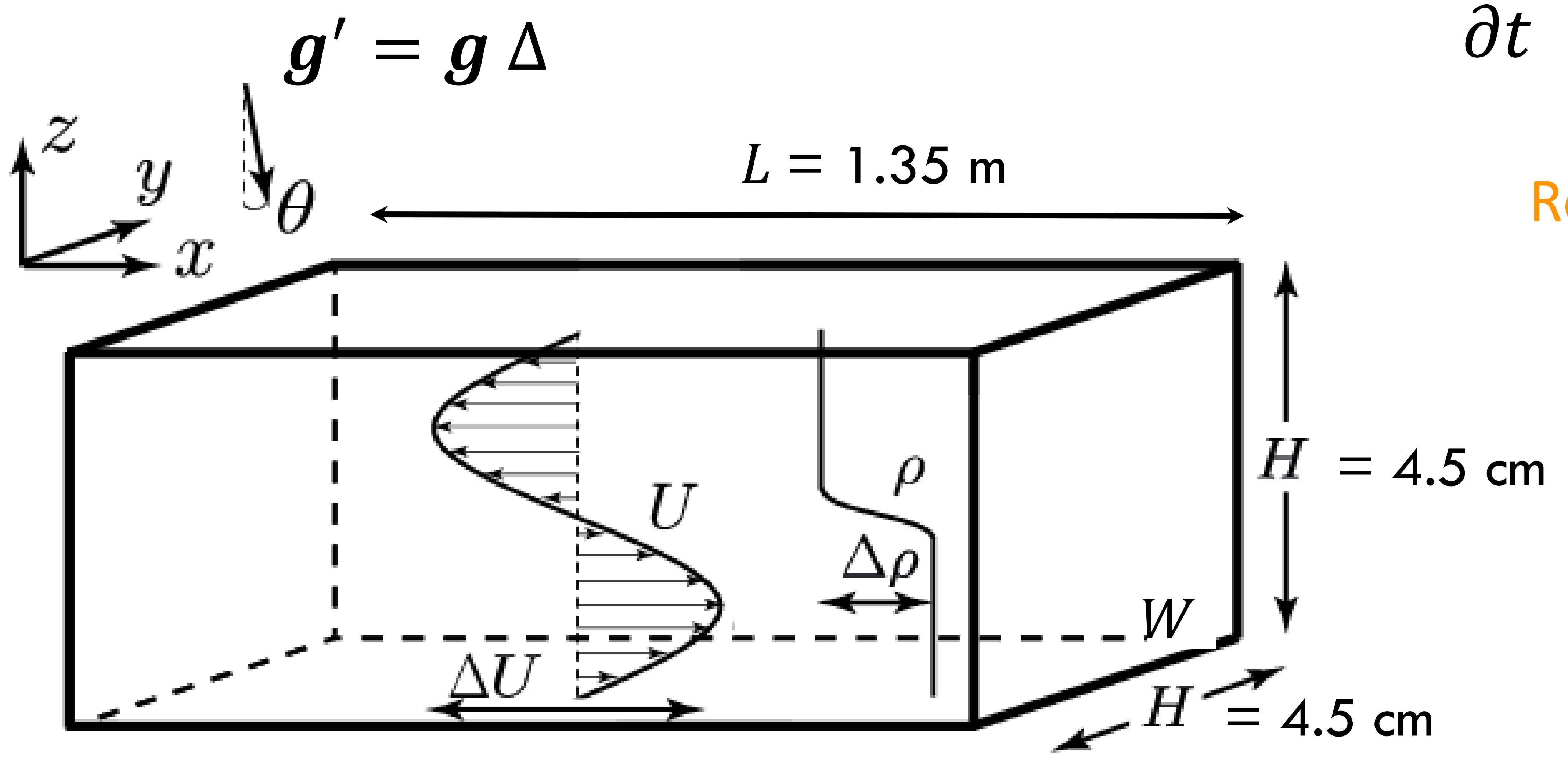
A sustained downslope gravity current in the lab





Focus on stratified shear flow
in a portion of the duct



Momentum:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} = -\boldsymbol{\nabla} p + \frac{1}{Re}\boldsymbol{\nabla}^2\boldsymbol{u} + Ri\,\rho\begin{pmatrix}\sin\theta\\ 0\\ -\cos\theta\end{pmatrix}$$

Tilt

Reynolds number $= \frac{\Delta U H}{4\nu}$

Richardson number $= \frac{\frac{g' H}{4}}{(\frac{\Delta U}{2})^2} \approx 0.15 - 0.5$

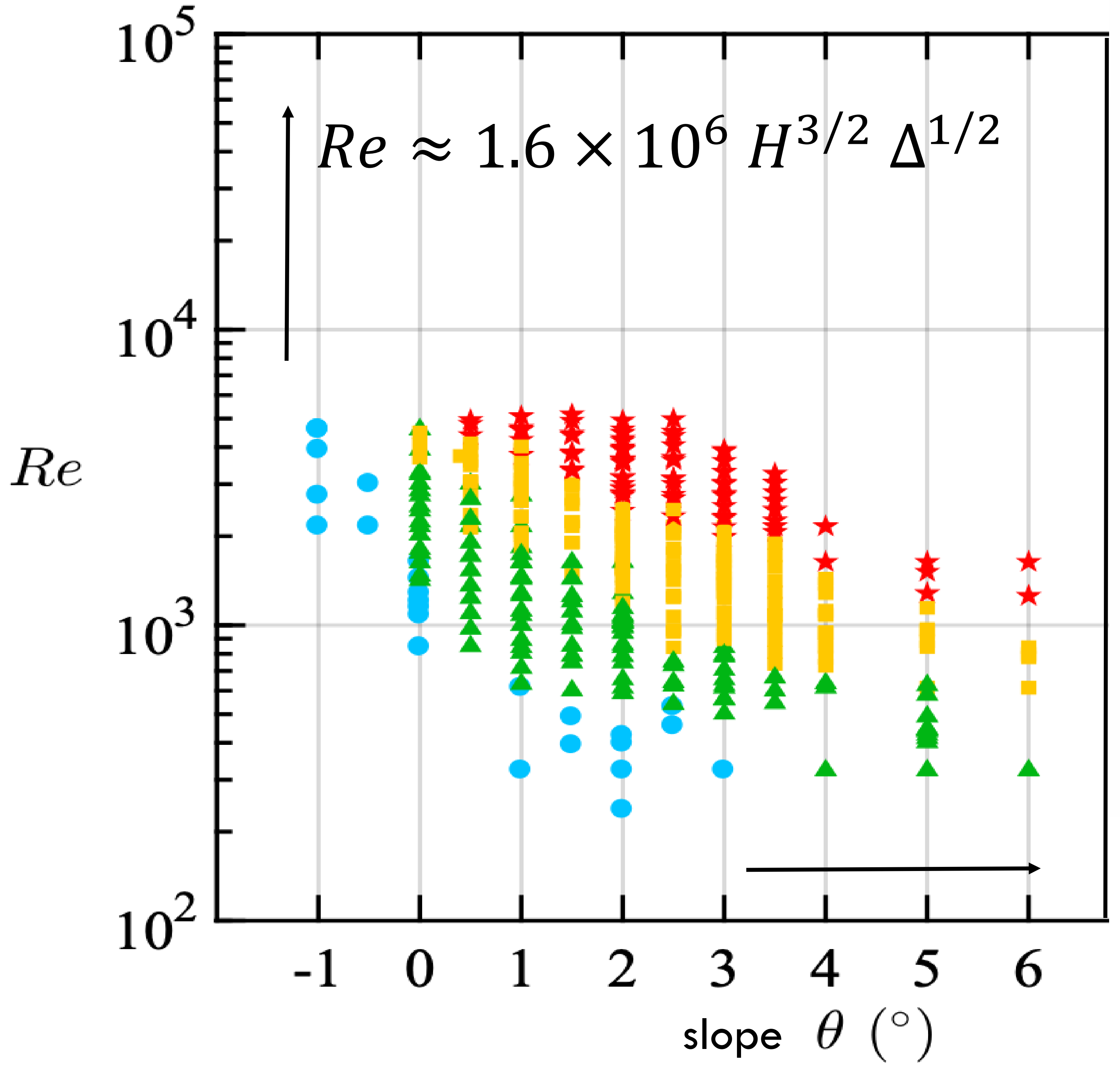
Buoyancy:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\rho = \frac{1}{Re\,Pr}\boldsymbol{\nabla}^2\rho$$

Prandtl number $= \frac{\nu}{\kappa} \approx 700$

Multiple regimes in parameter space, relevant to geophysical flows

SID Regime Diagram
(subset of >1000 experiments)



Turbulent dissipation rate

$$\varepsilon \approx 2.2 H^{1/2} \Delta^{3/2} \theta \quad \mathrm{m^2\ s^{-3}}$$

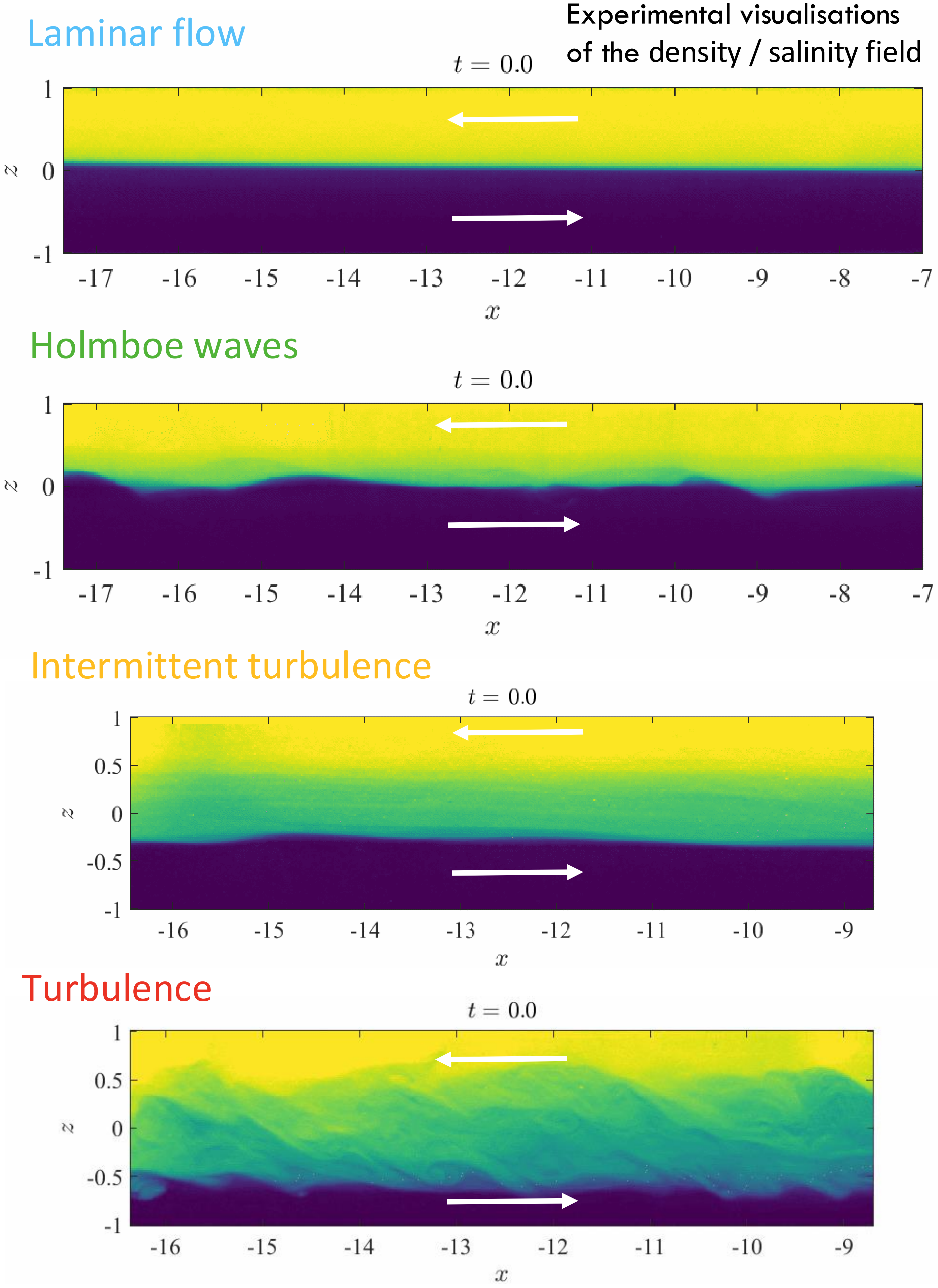
Buoyancy frequency

$$N \approx 2.6 H^{-1/2} \Delta^{1/2} \quad \mathrm{rad/s}$$

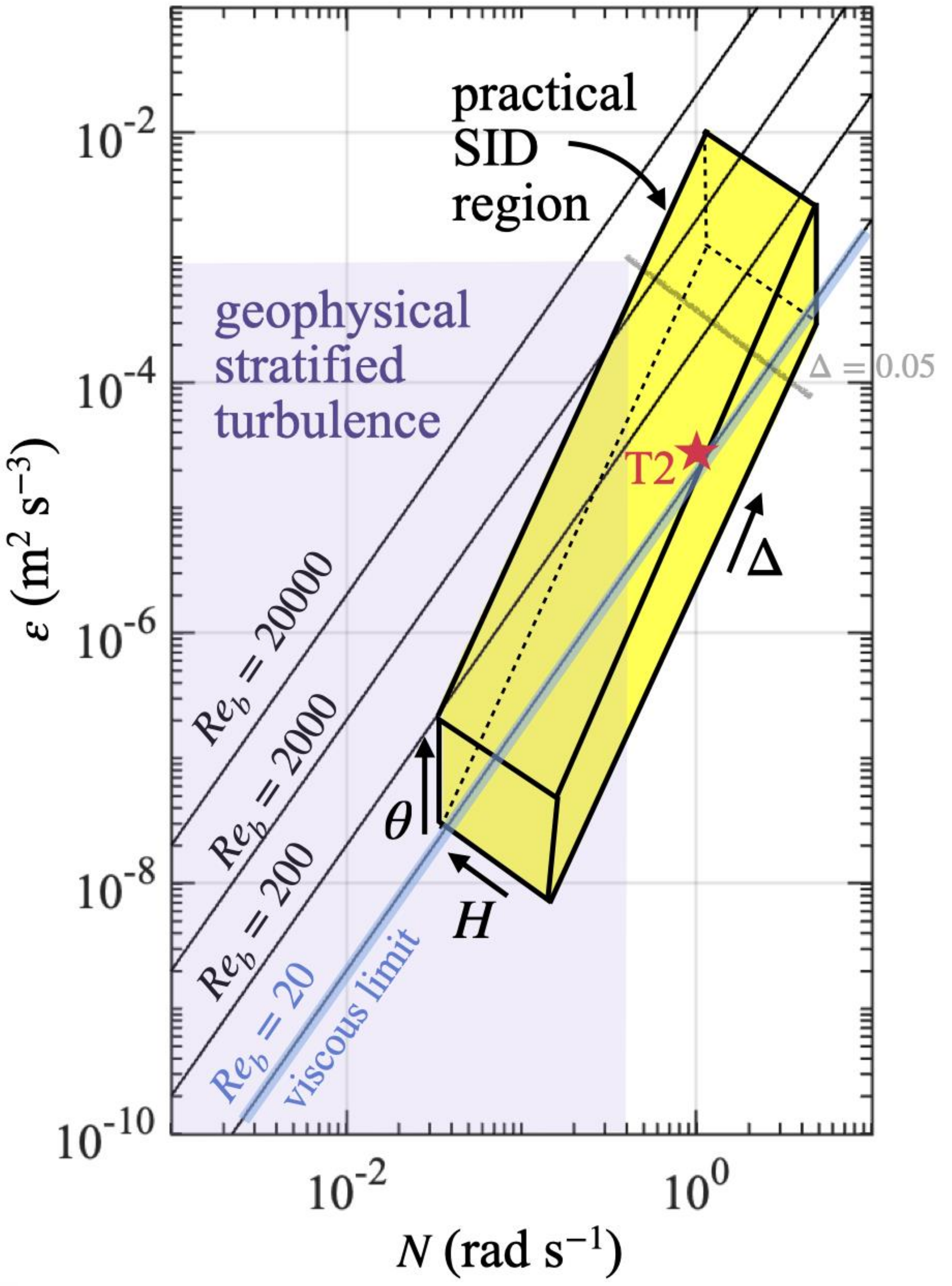
Buoyancy Reynolds number

$$Re_b \approx 0.2\ Re\ \theta$$

Experimental visualisations of the density / salinity field

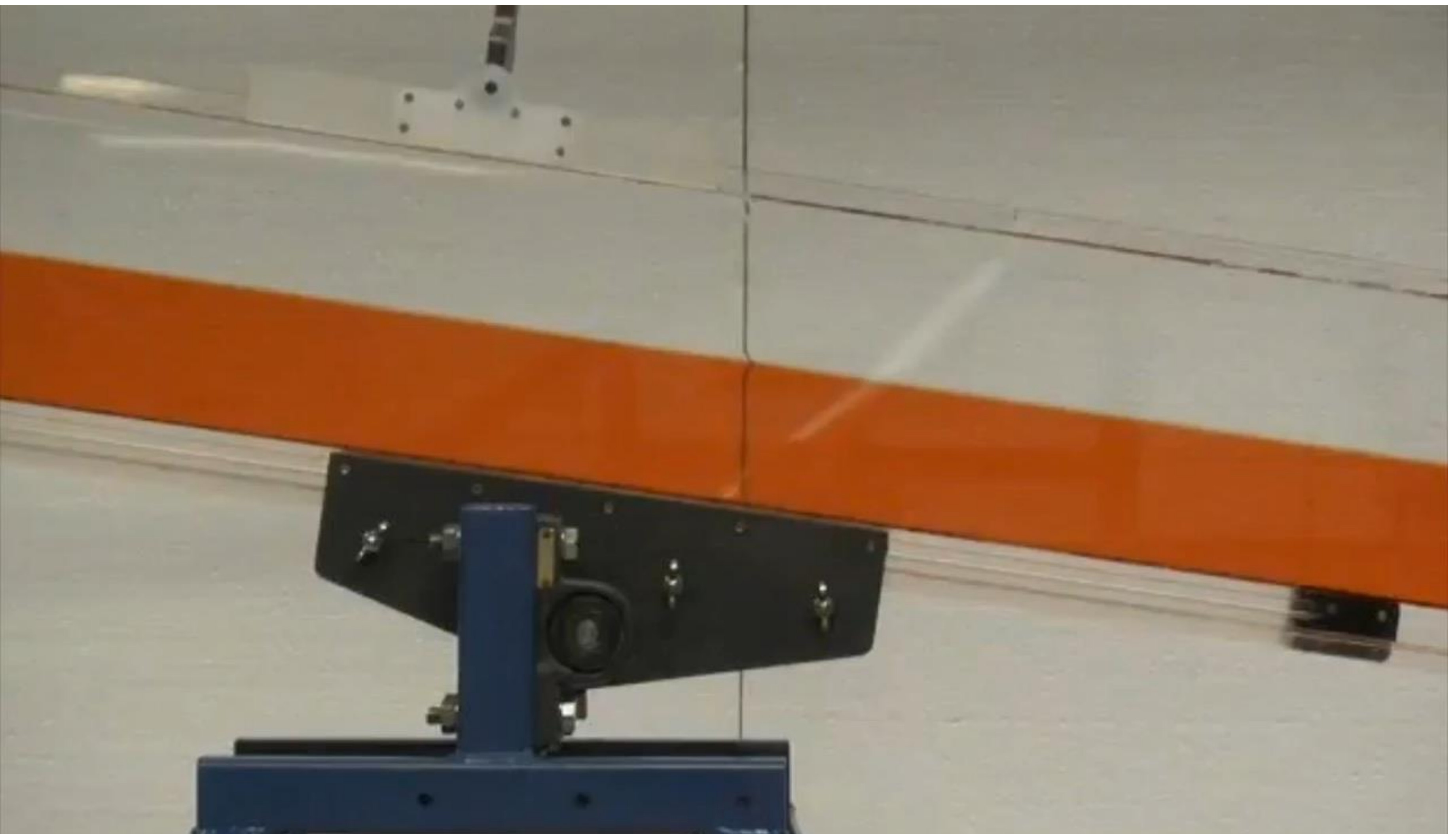


Review:
Lefauve, Comptes Rendus Physique (2024)

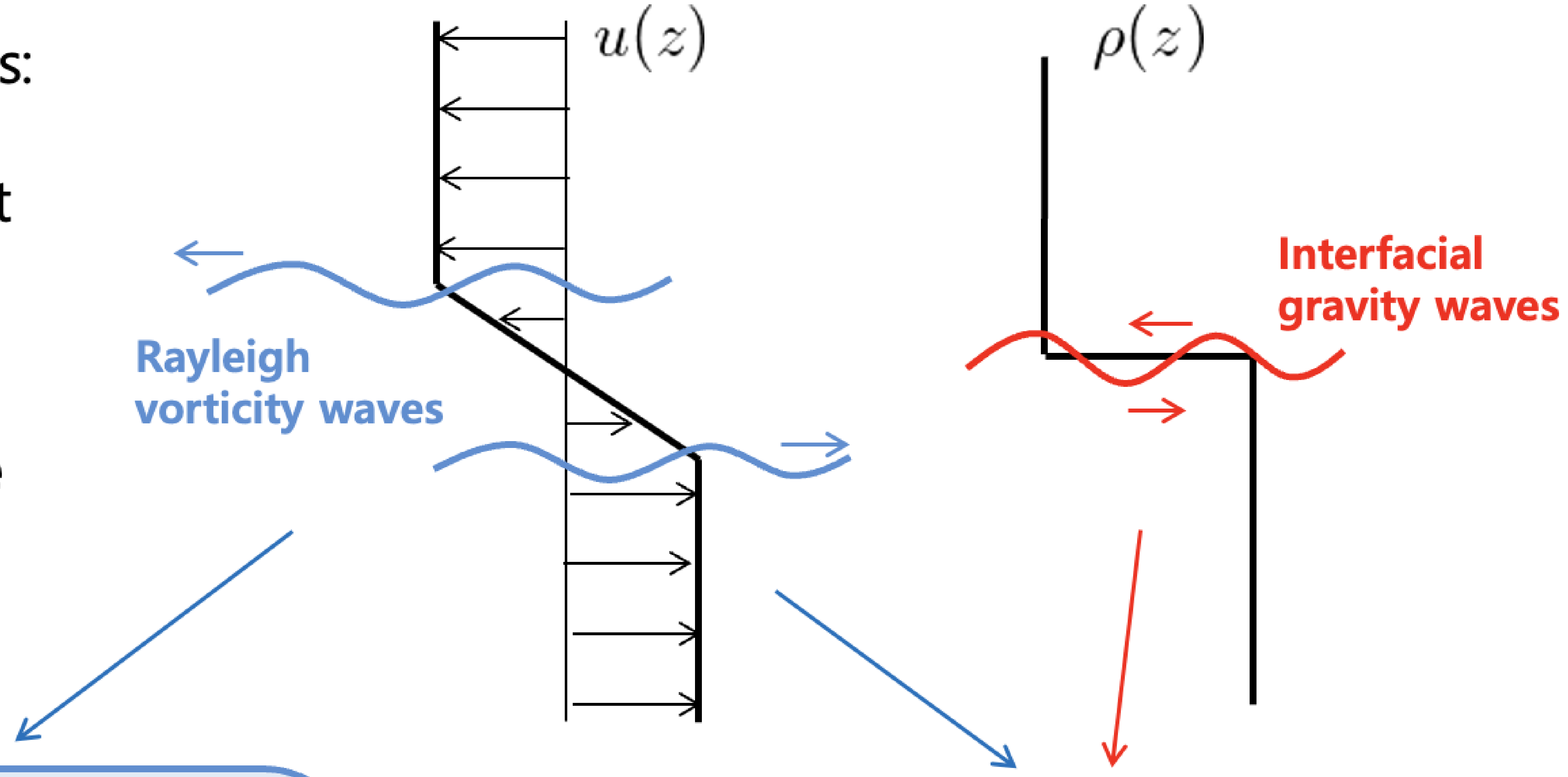


Today: focus on the Holmboe wave regime

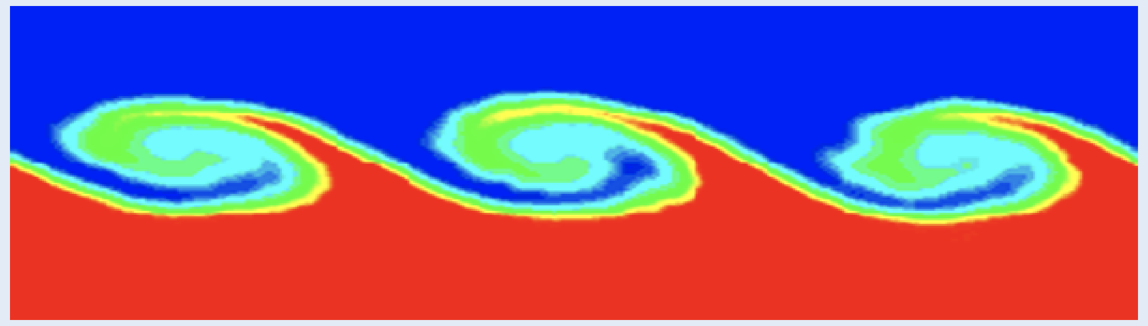
Long-lived Holmboe waves
≠
transient Kelvin-Helmholtz turbulence



- Idealised 1D profiles:
- Dispersive waves: at some λ, phase speeds are equal
  → **waves interact, 'lock' and become unstable**

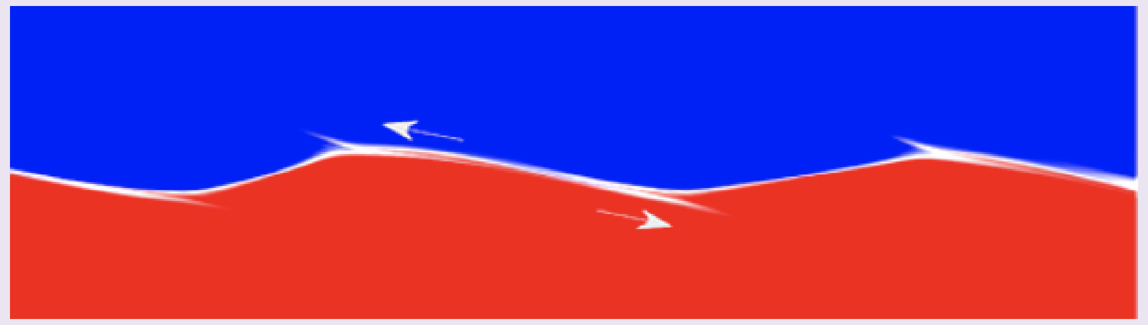


Kelvin-Helmholtz instability (stationary)



suppressed by buoyancy forces

Holmboe instability (travelling)

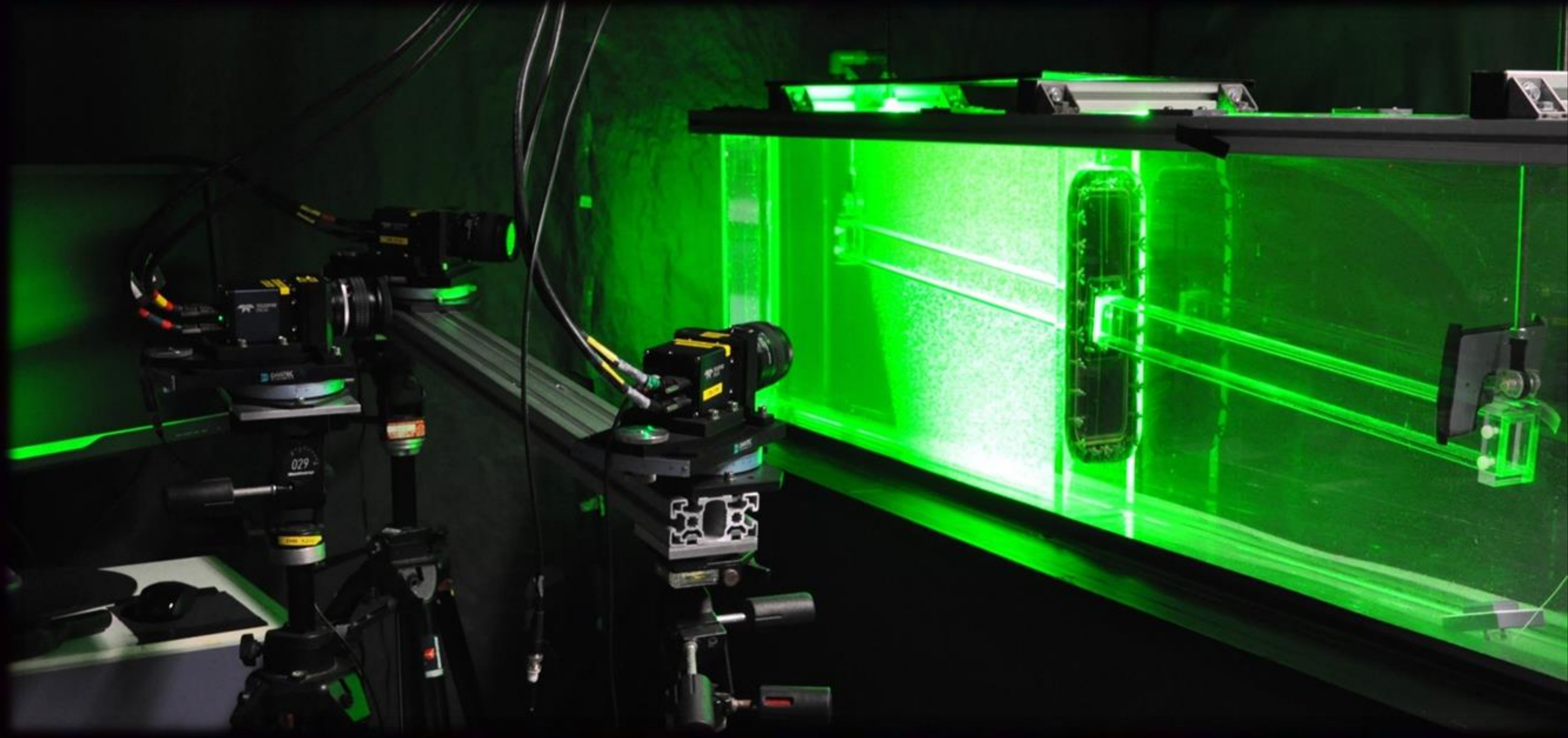


enhanced by buoyancy forces

- Show **potential of a physics-informed neural network** to augment experiments
- Start with a relatively slow, **non-turbulent dataset**
- Illustrate the **physical insights gained** by augmented data

Optical flow measurements

Record 3 x 50 GB of raw data per experiment



Holmboe waves

small (30 μm) particles to track the flow

→ **2D** velocity field

(Particle Image Velocimetery = PIV)

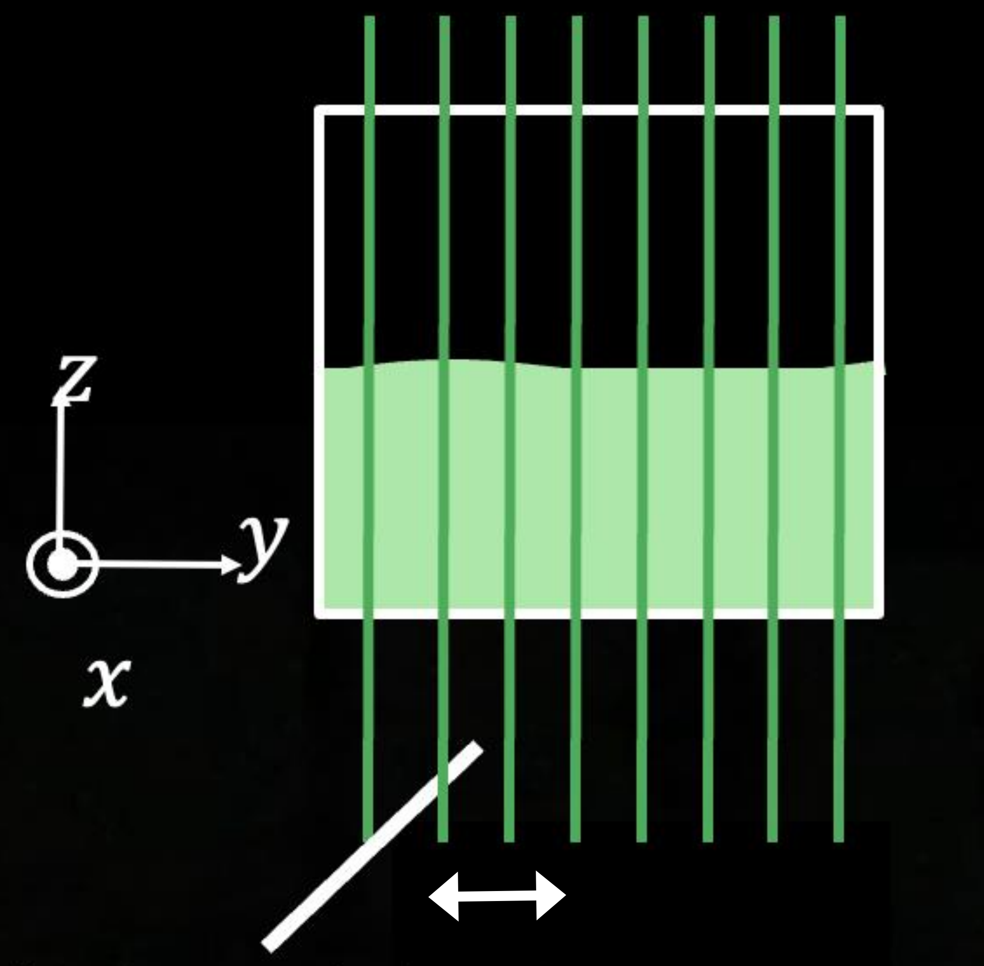
simultaneous

fluorescent dye to tag the denser water

→ **2D** density field

(Laser-Induced Fluorescence = LIF)

Volumetric measurements



3D velocity and density
at vector resolution

500 x 40 x 100 x 300
in x y z t

Step change in last 5-10 years!

Partridge, Lefauve & Dalziel, Meas. Sci. Tech. (2019)

Video credits: Xianyang Jiang

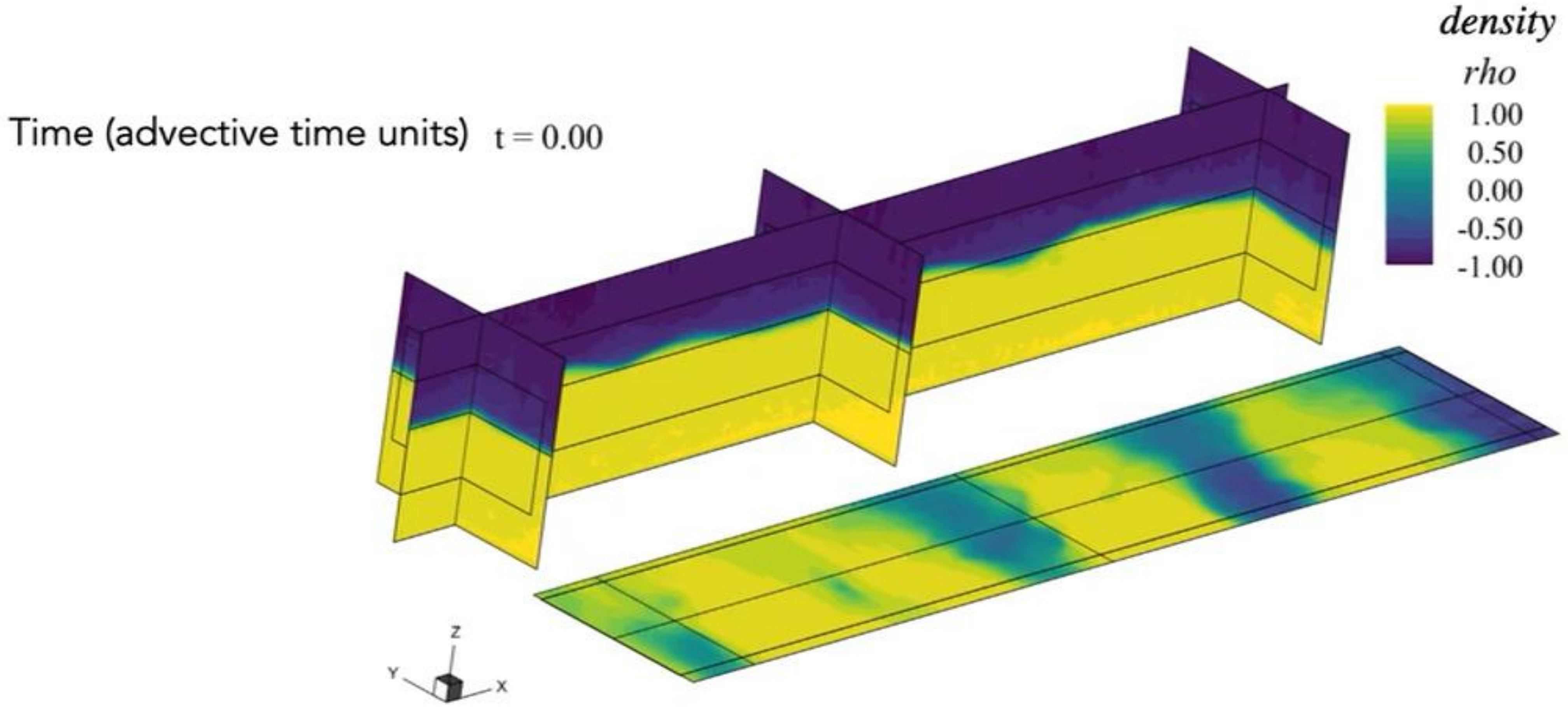
An example of 3D Holmboe dataset

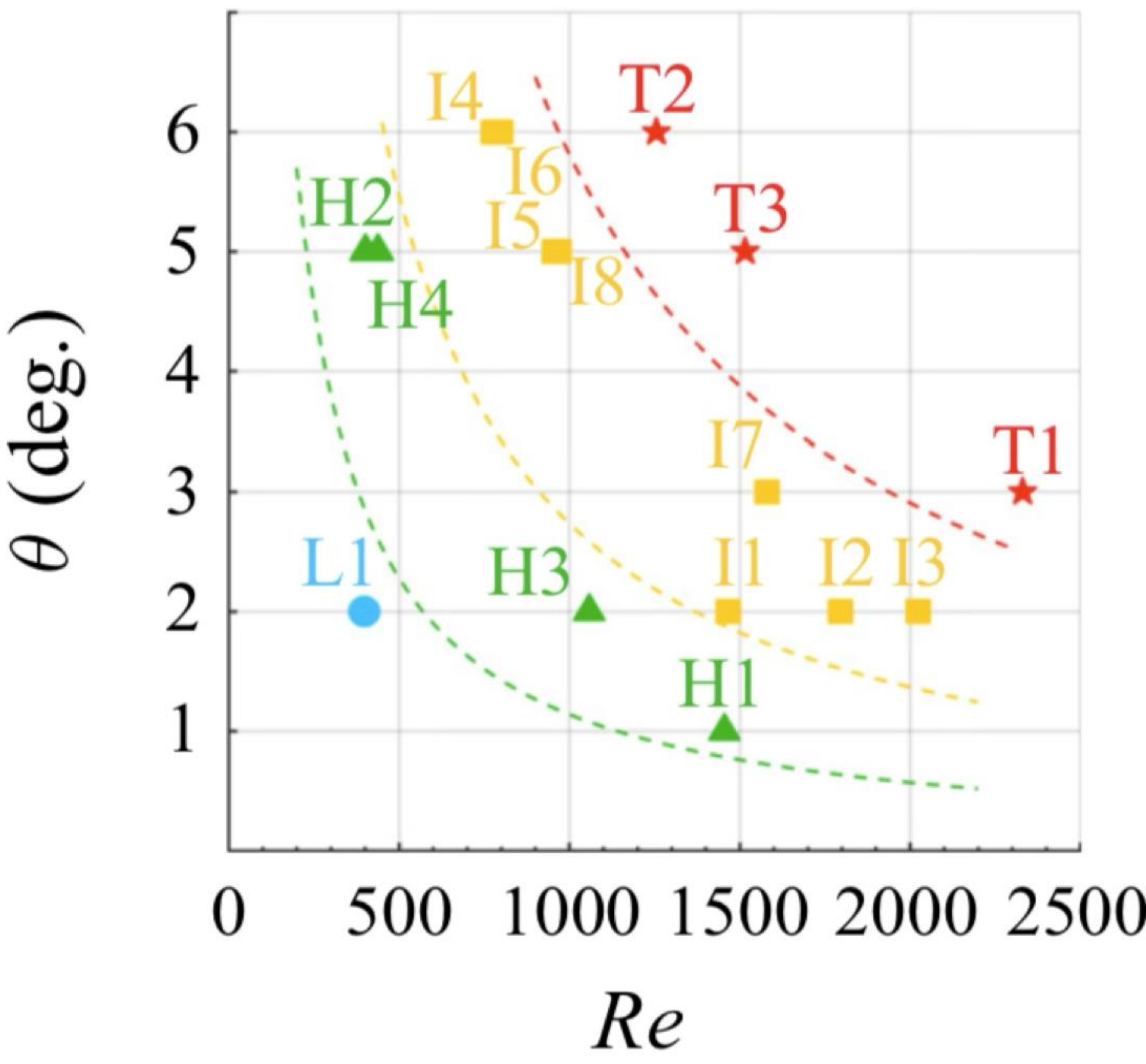
Note: $\nabla \cdot \boldsymbol{u} = \mathbf{0}$ imposed at all times with variational method

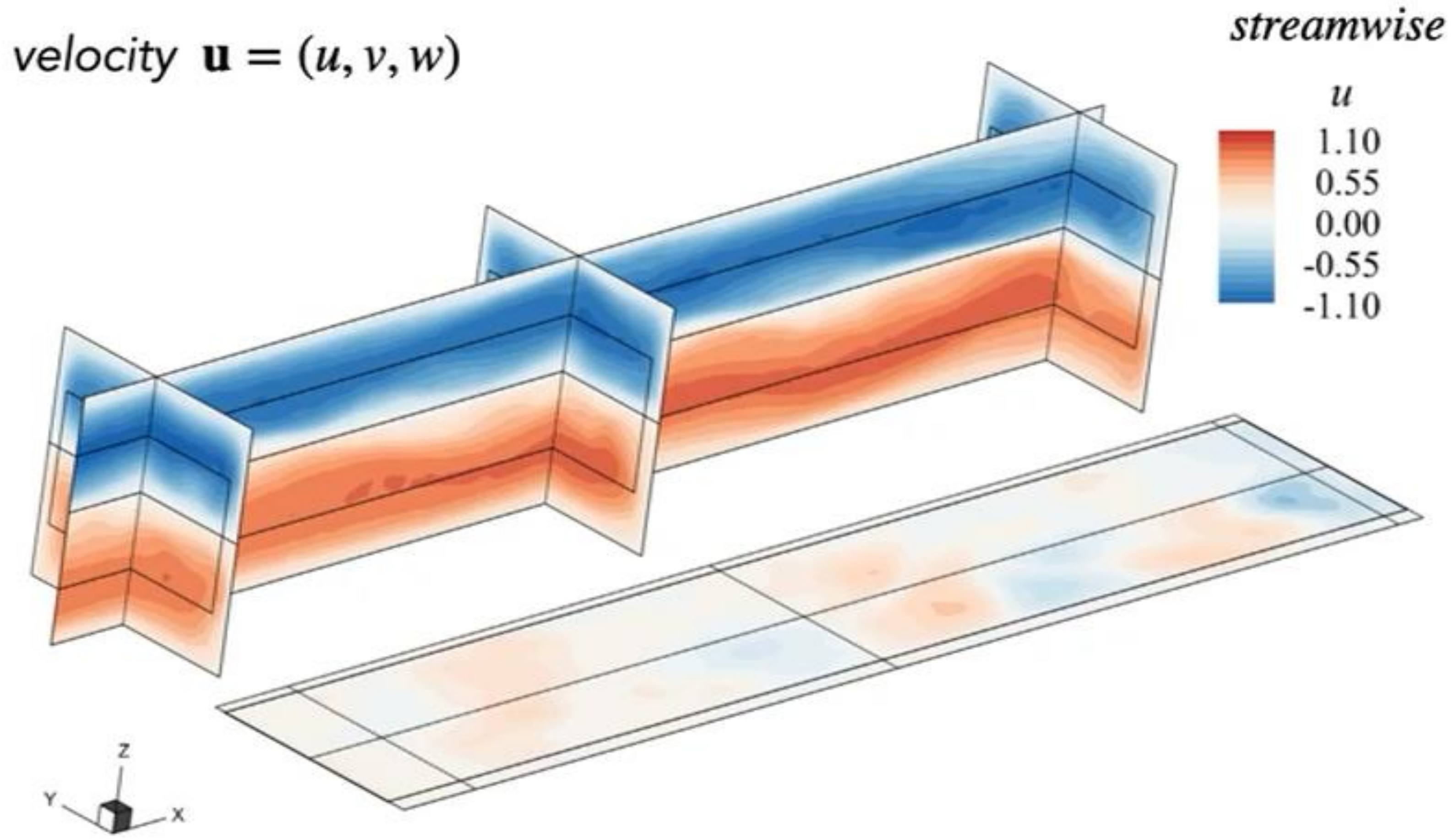
(Wang, Exp. Fluids 2017)

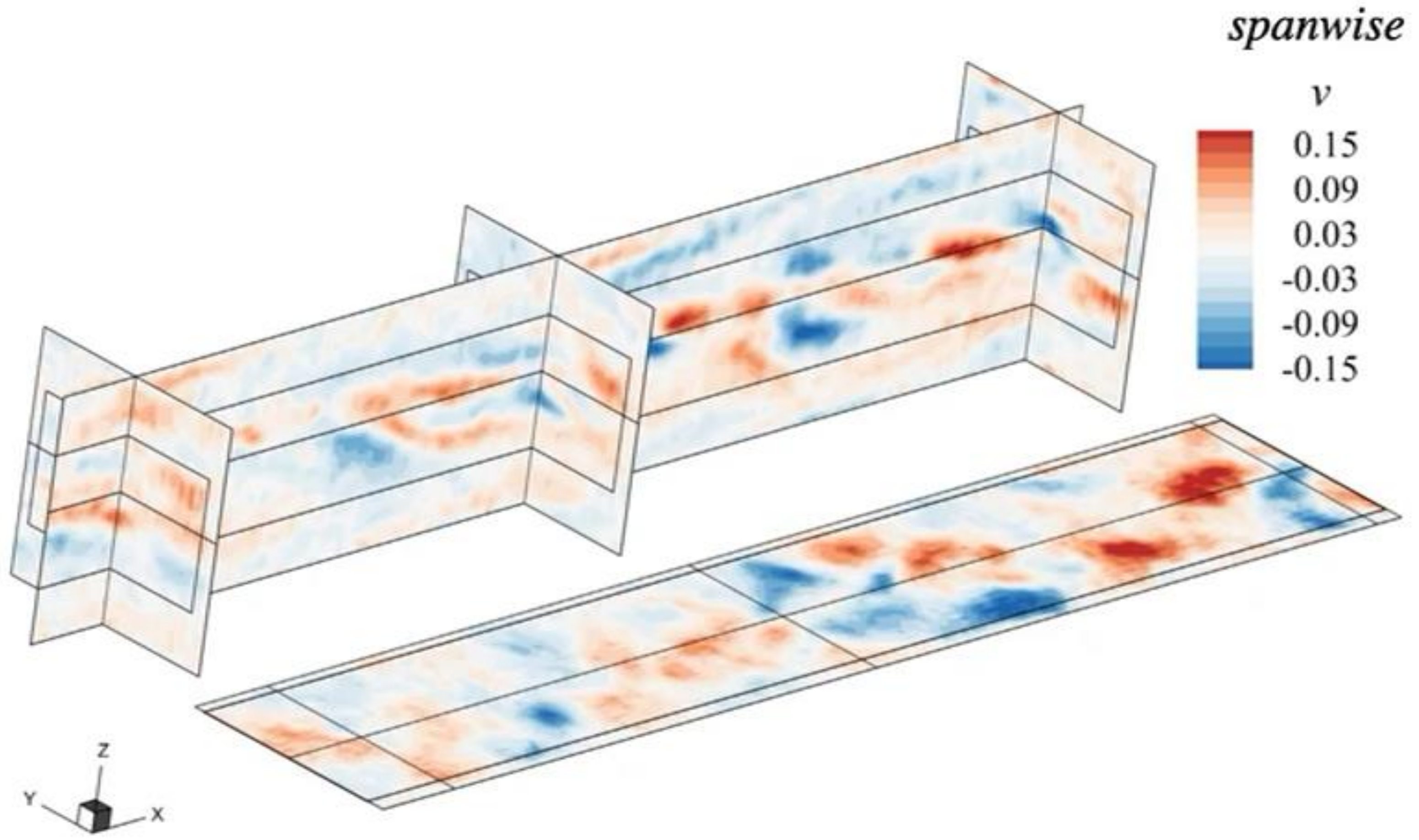
Limitations:

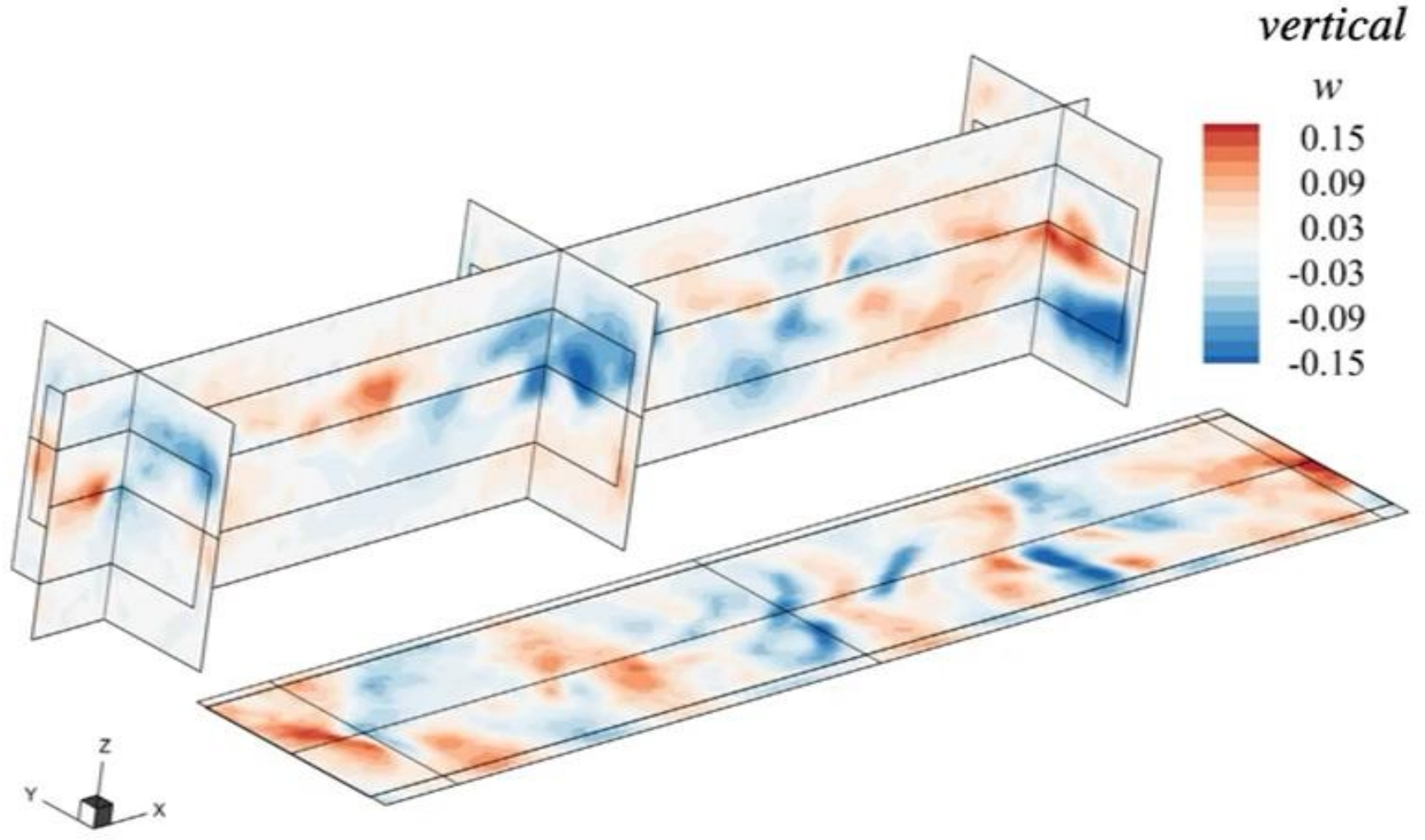
- Signal-to-noise ratio (esp. for gradients)
- Scanning along y (distortion)
- No pressure field











All 16 datasets are freely available

Source: Lefauve, A. & Linden, P.F. 2022 Research data supporting "Experimental properties of continuously-forced, shear-driven, stratified turbulence" [Dataset]. doi.org/10.17863/CAM.75370.

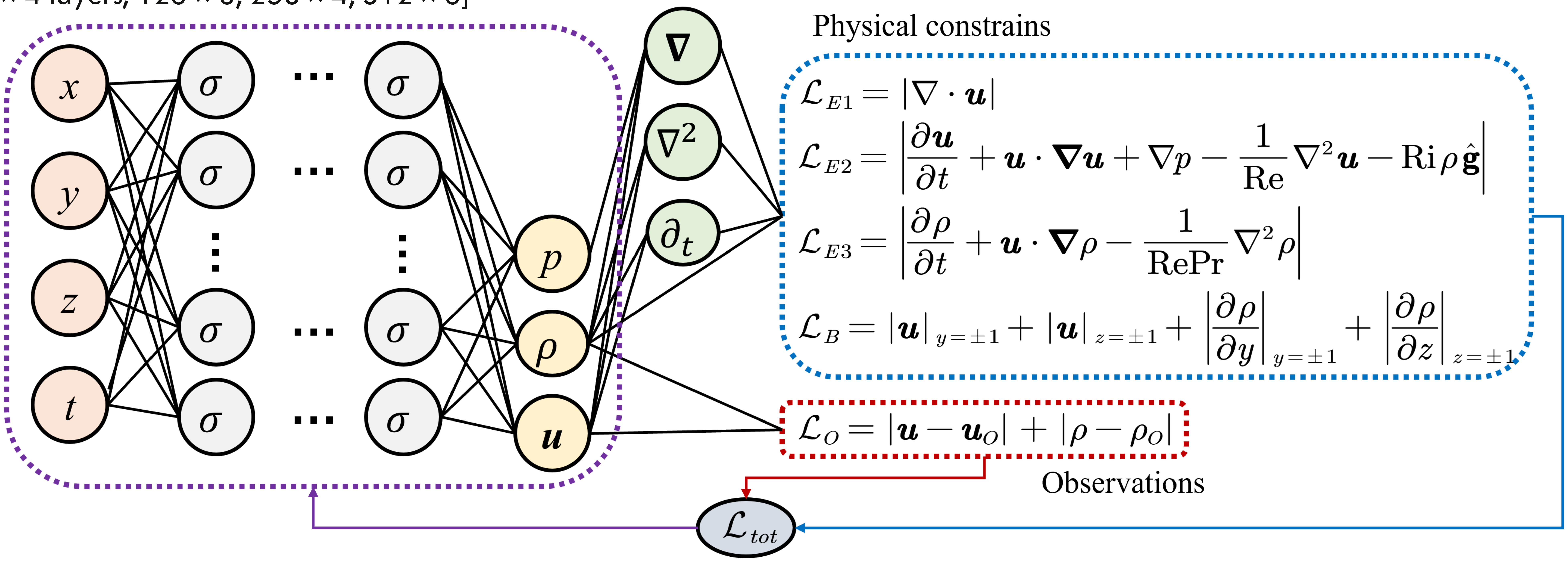
A Physics Informed Neural Network (PINN) for stratified flows

- Original PINN idea: integrate observations and physical laws to approximate unknown field (Raissi et al., J. Comp. Phys., 2019)
- PINNs showed promise for **experimental fluid mechanics** (Cai et al. 2021; Wang, Liu & Wang 2022a; Fan et al. 2023)
- Here first application to **3D stratified flow data**

Fully connected deep neural network composed of 14 layers with an increasing number of artificial neurons [64 × 4 layers, 128 × 3, 256 × 4, 512 × 3]

Automatic differentiation (Baydin et al. 2018)

Training on an individual experimental dataset (1 million grid points per volume, ~10-100 volumes)
Total ~ 100 GPU hours on NVIDIA A100 per experiment

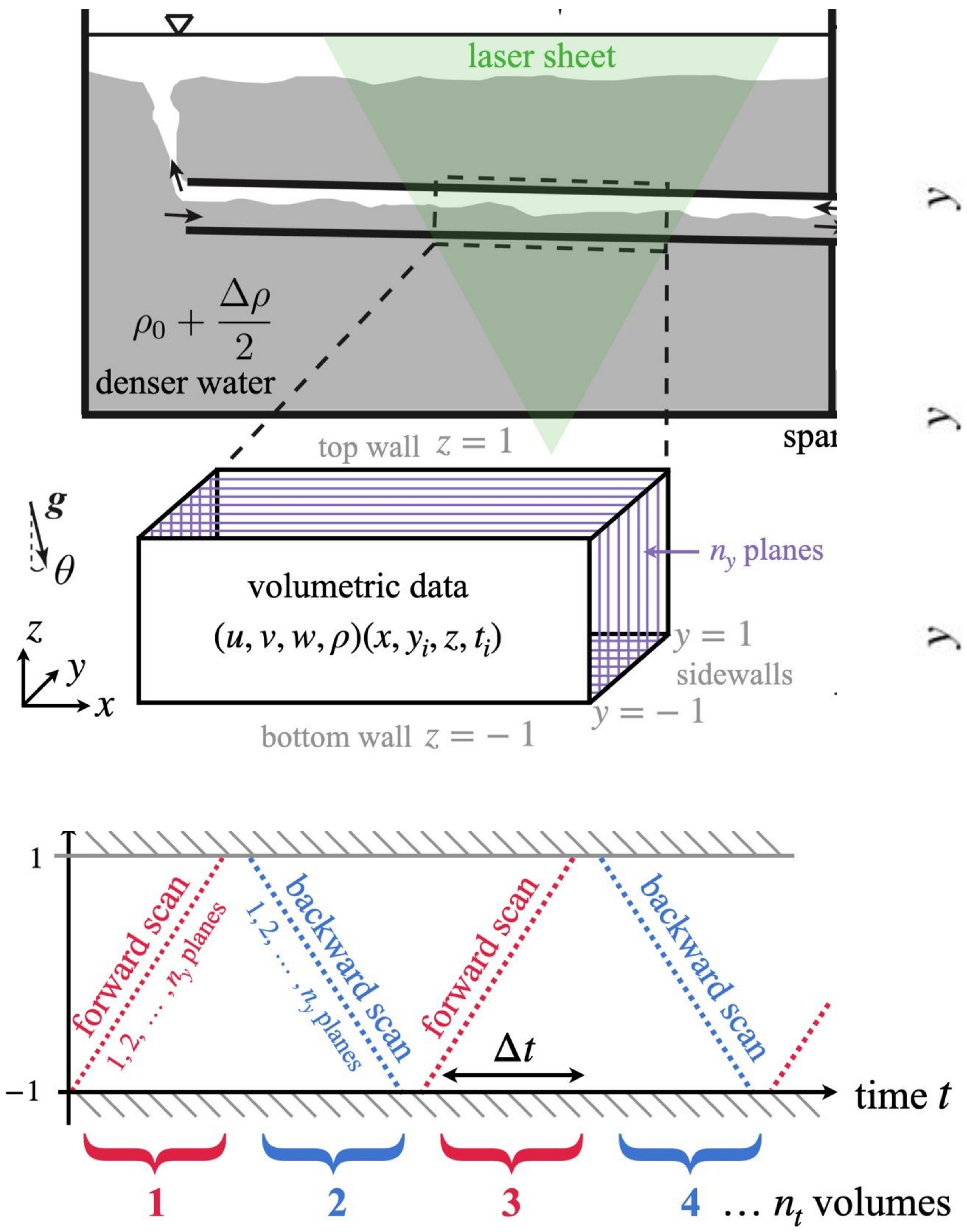


ADAM algorithm to minimise total loss (Kingma & Ba 2014)

Result 1: the PINN undoes the spanwise distortion of scanned data

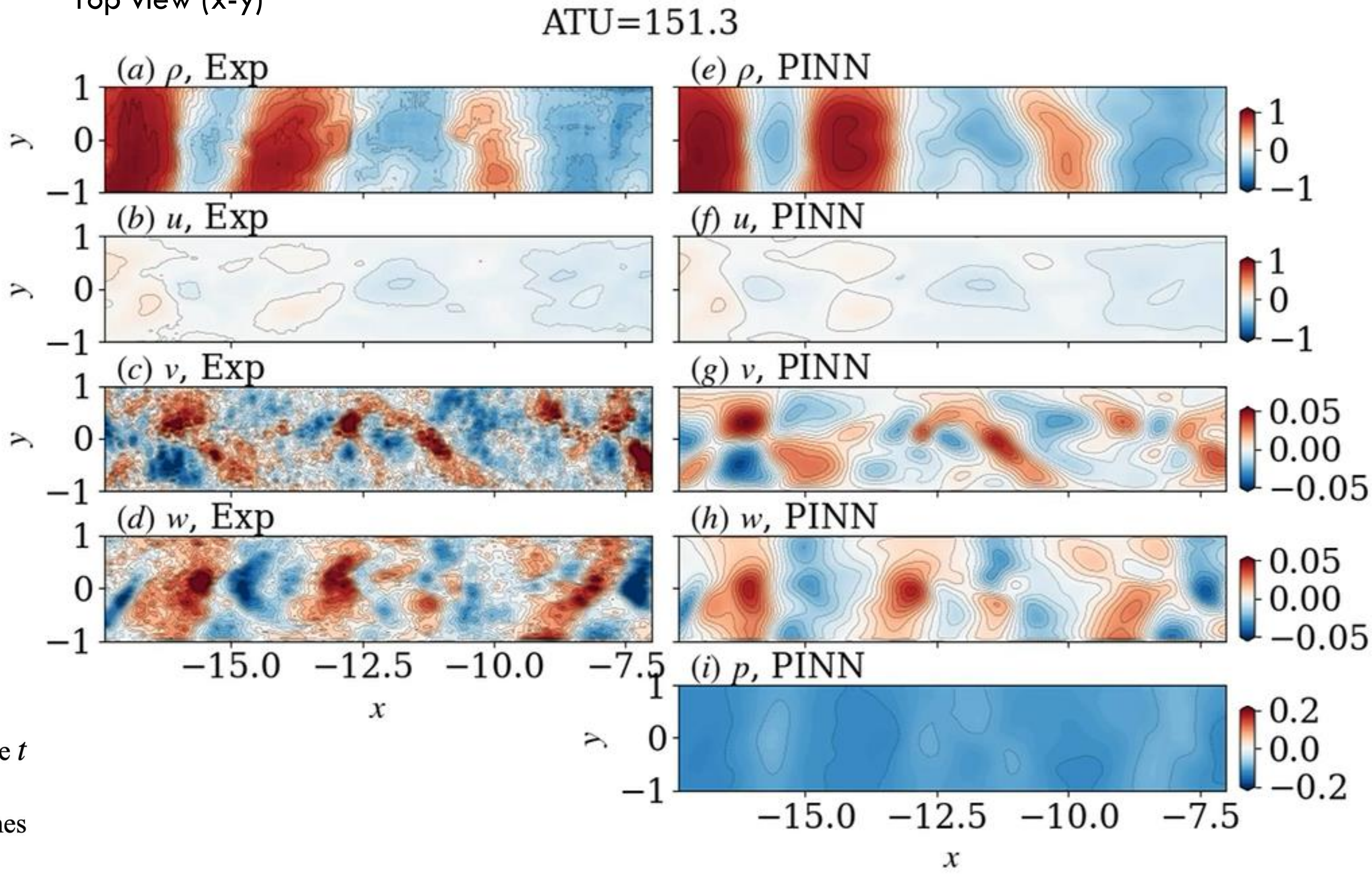
The PINN is fed the **real scanned data**

$(\boldsymbol{u}, \rho)(x, y_i, z, t_i)$



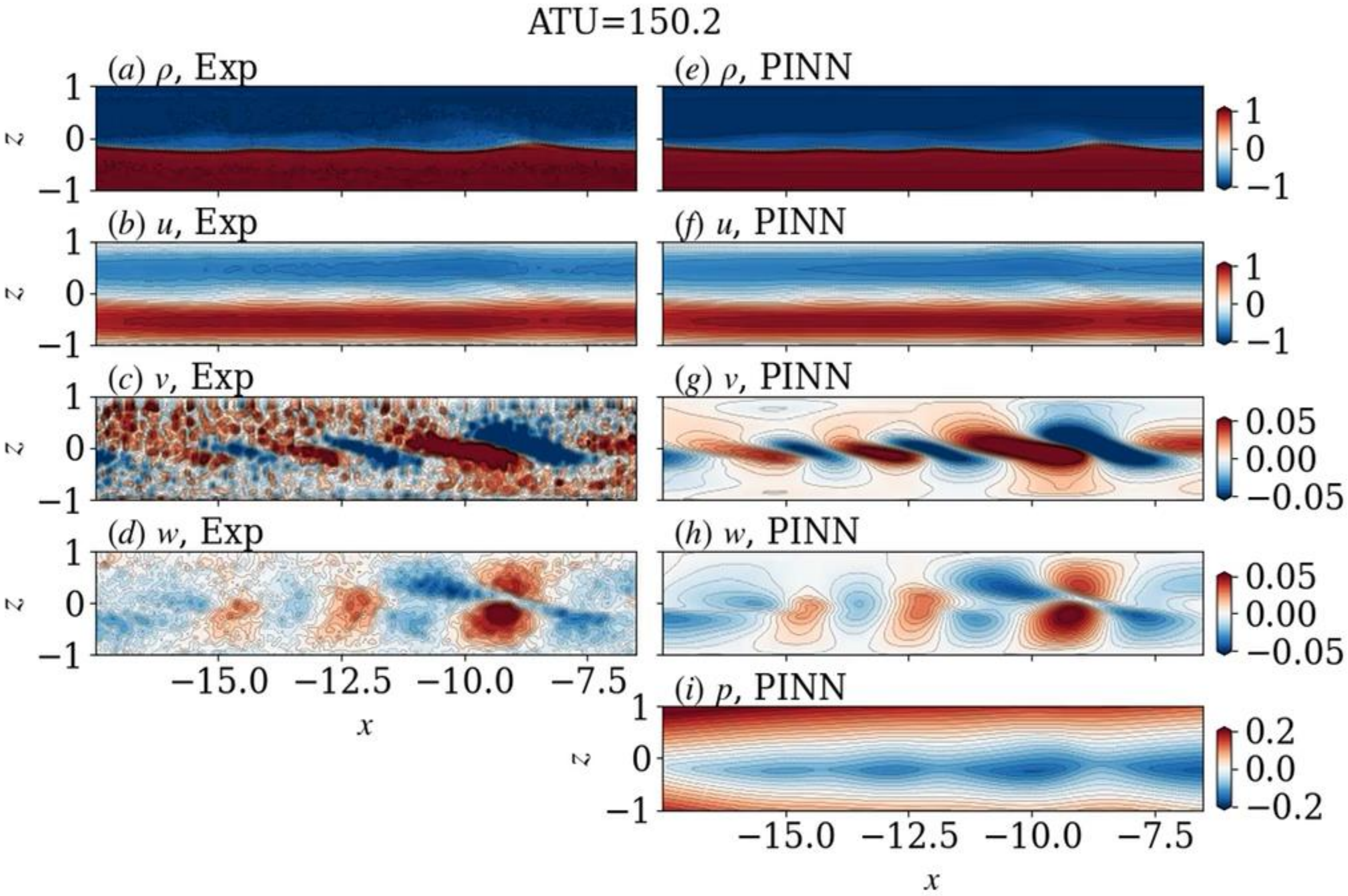
The reconstruction outputs the instantaneous volumes

Top view (x-y)



Result 2: Reduced noise and improved spatio-temporal resolution

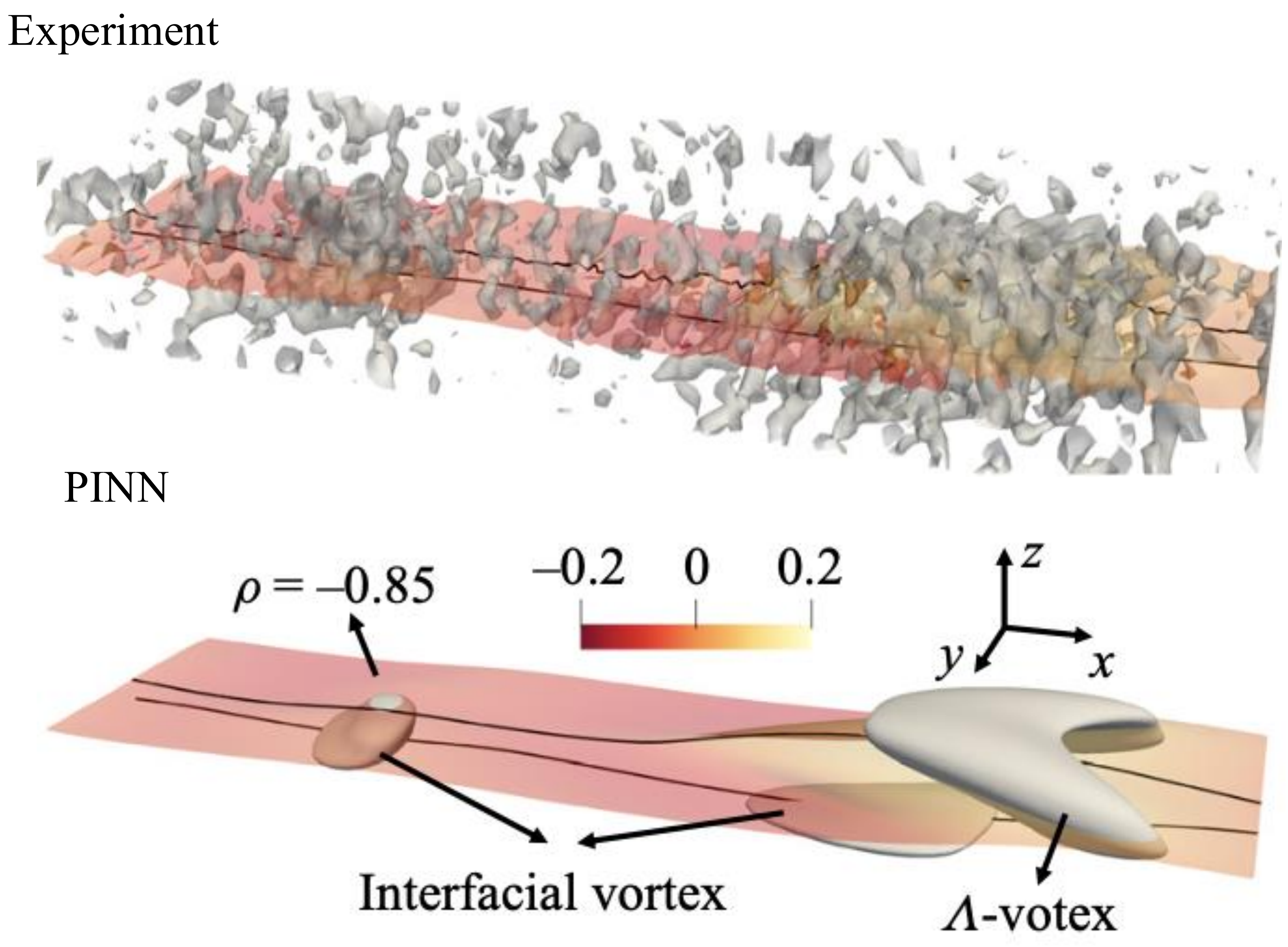
Side view (x-z)



Better identification **of 3D coherent vortical structures**

Example: iso-surfaces of Q-criterion, sensitive to noise

$$Q = (1/2)(||\boldsymbol{W}||^2 - ||\boldsymbol{E}||^2)$$

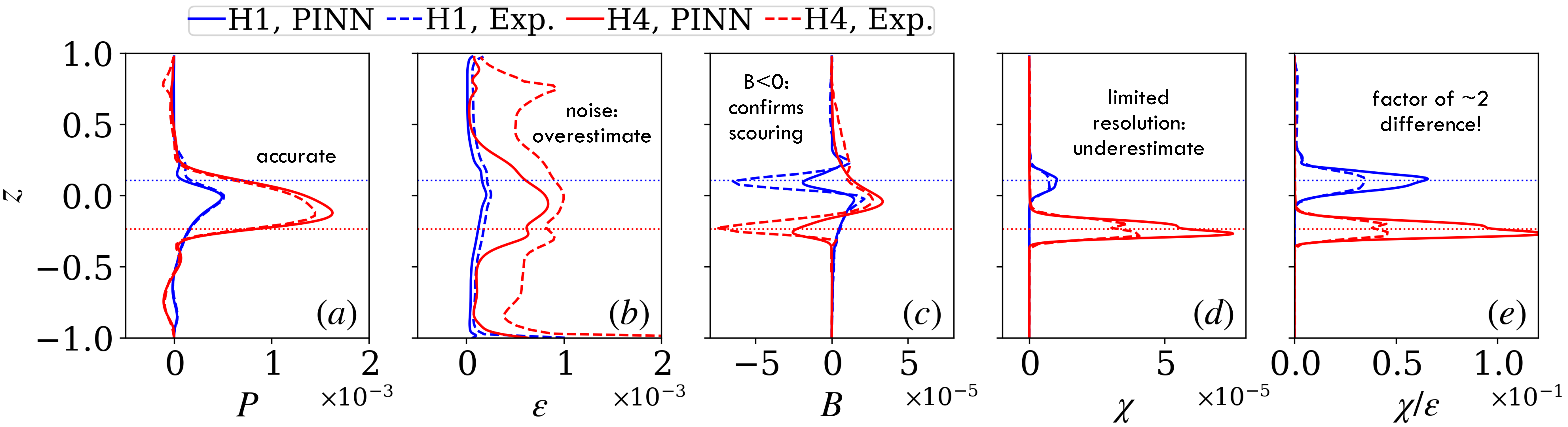


Precursors to hairpin vortices in the turbulent regime

Jiang et al. *J Fluid Mech.* (2022)

Result 2: Reduced noise and improved spatio-temporal resolution

Improved closure of the turbulent energy budgets



Production TKE dissipation Buoyancy flux Scalar dissipation Mixing efficiency

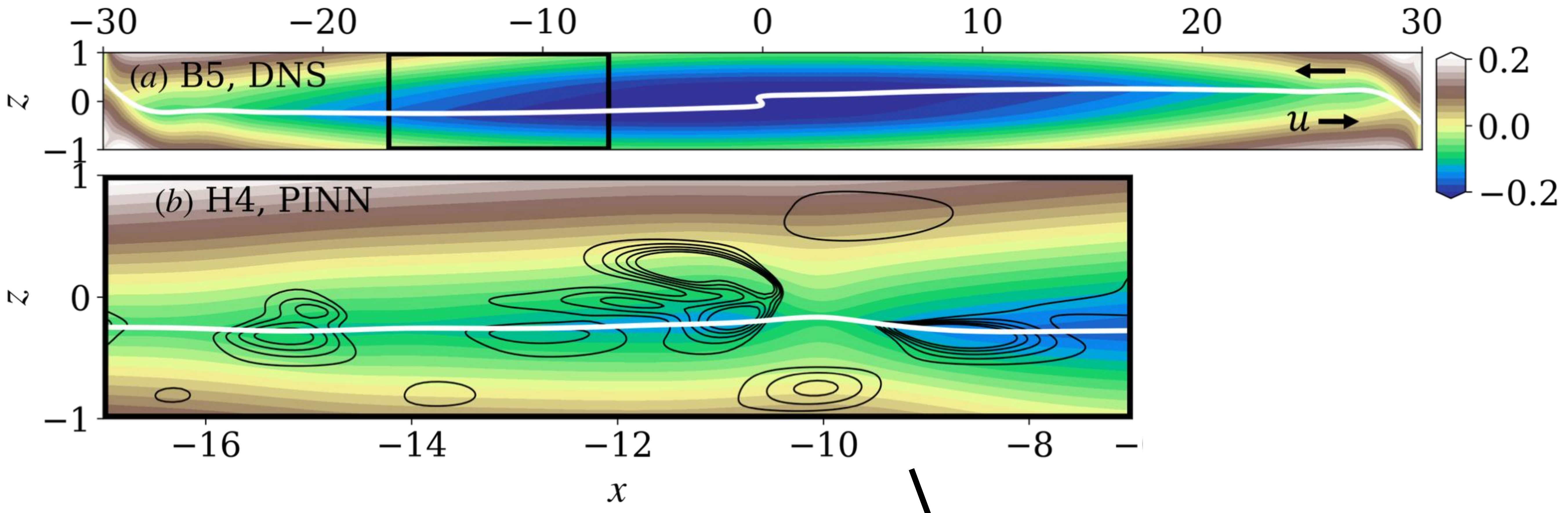
$$P \equiv -\langle u'v'\partial_y \overline{u} + u'w'\partial_z \overline{u}\rangle, \quad \epsilon \equiv \frac{2}{\mathrm{Re}}\langle ||\boldsymbol{E}'||^2\rangle, \quad B \equiv \mathrm{Ri}\langle w'\rho'\rangle, \quad \chi \equiv \frac{\mathrm{Ri}}{\mathrm{Re\ Pr}}\langle |\boldsymbol{\nabla}\rho'|^2\rangle,$$

Result 3: Access to the latent pressure field

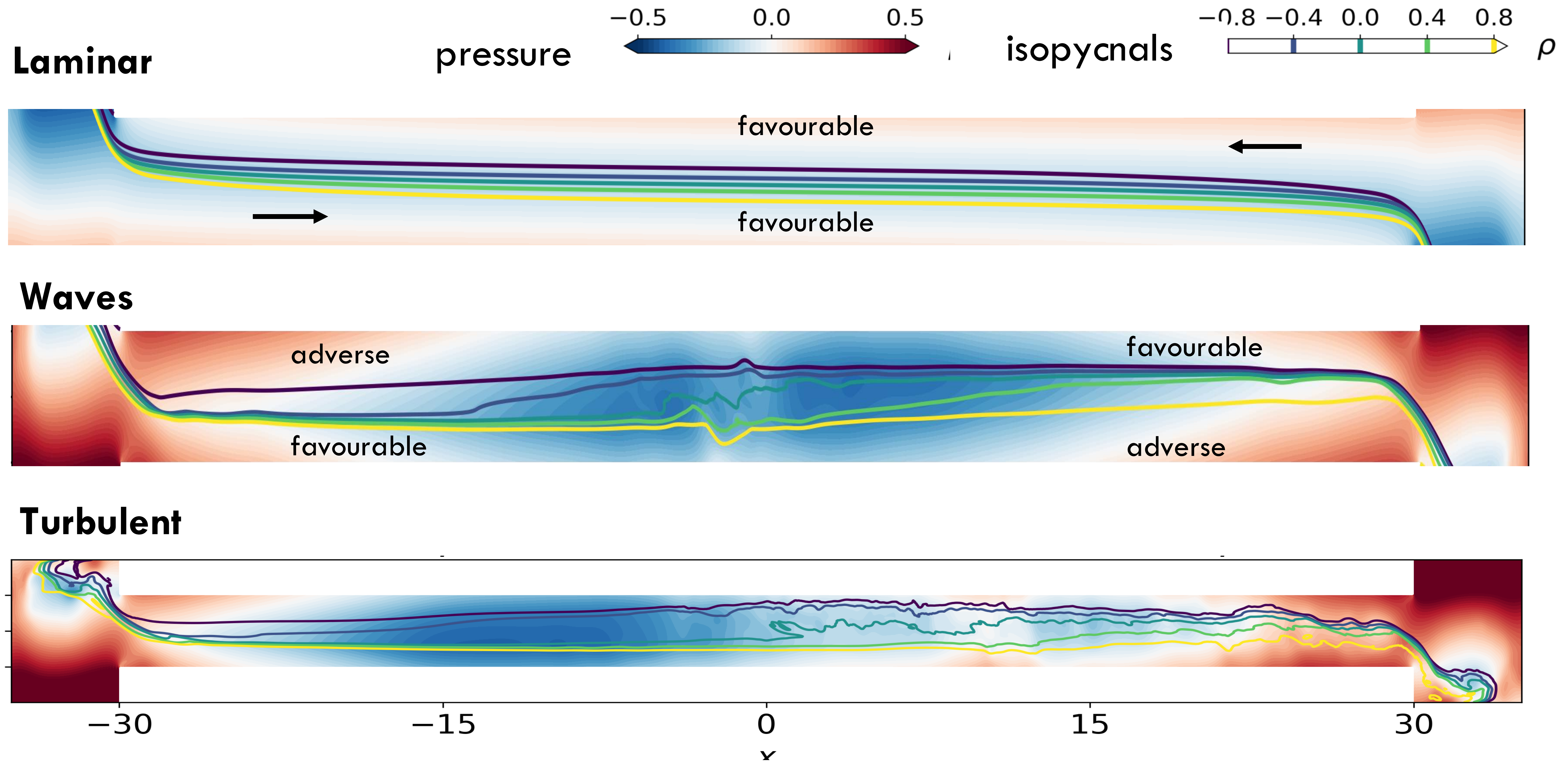
Previous Direct Numerical Simulations (DNS) highlight the role of the internal **pressure field**

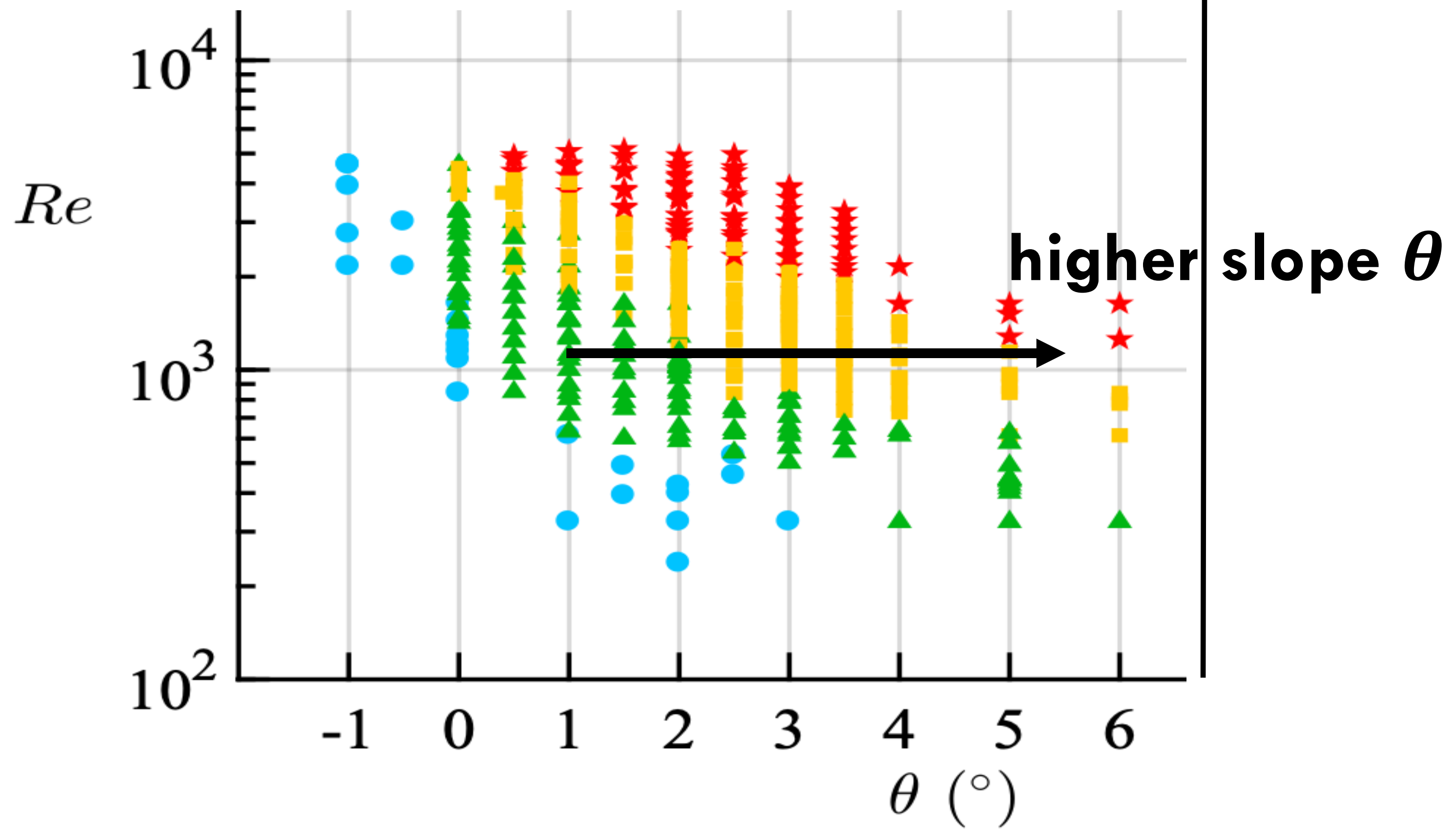
Zhu et al., J. Fluid Mech. (2023)

At higher slope θ, an **adverse pressure gradient** creates unstable waves and turbulence



The **PINN pressure in the measured window is consistent with DNS** in a similar regime (but different Pr)

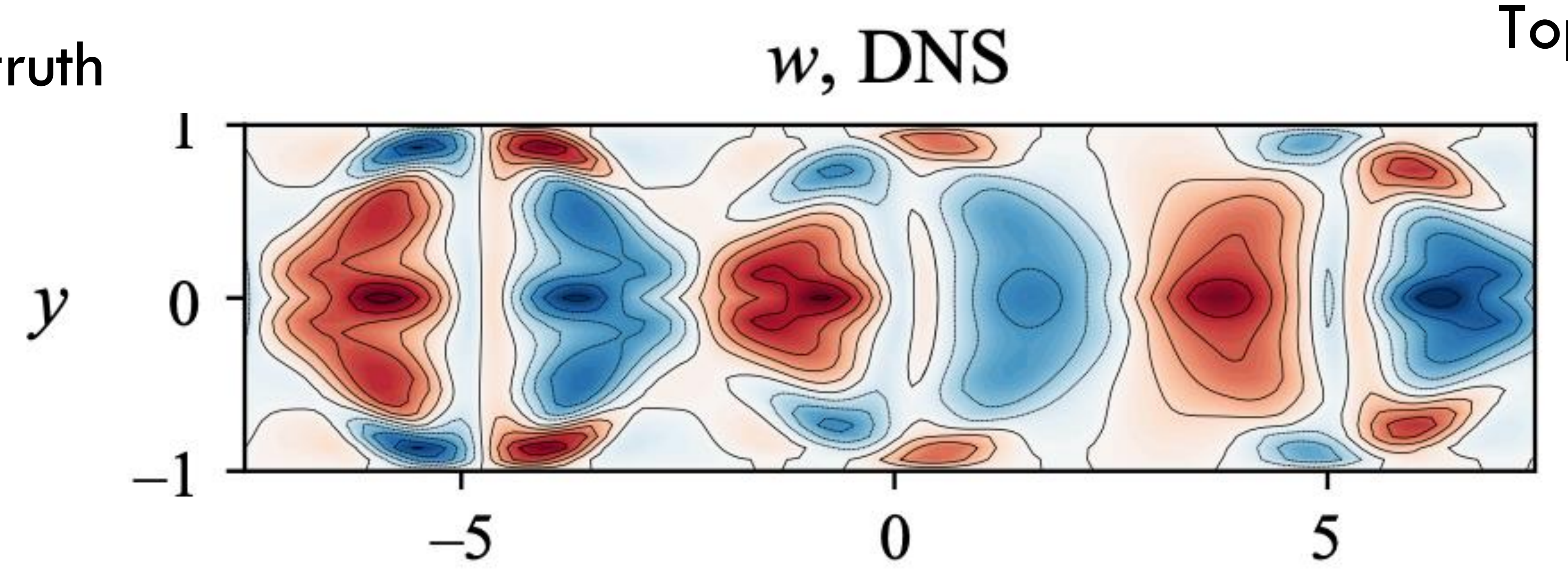


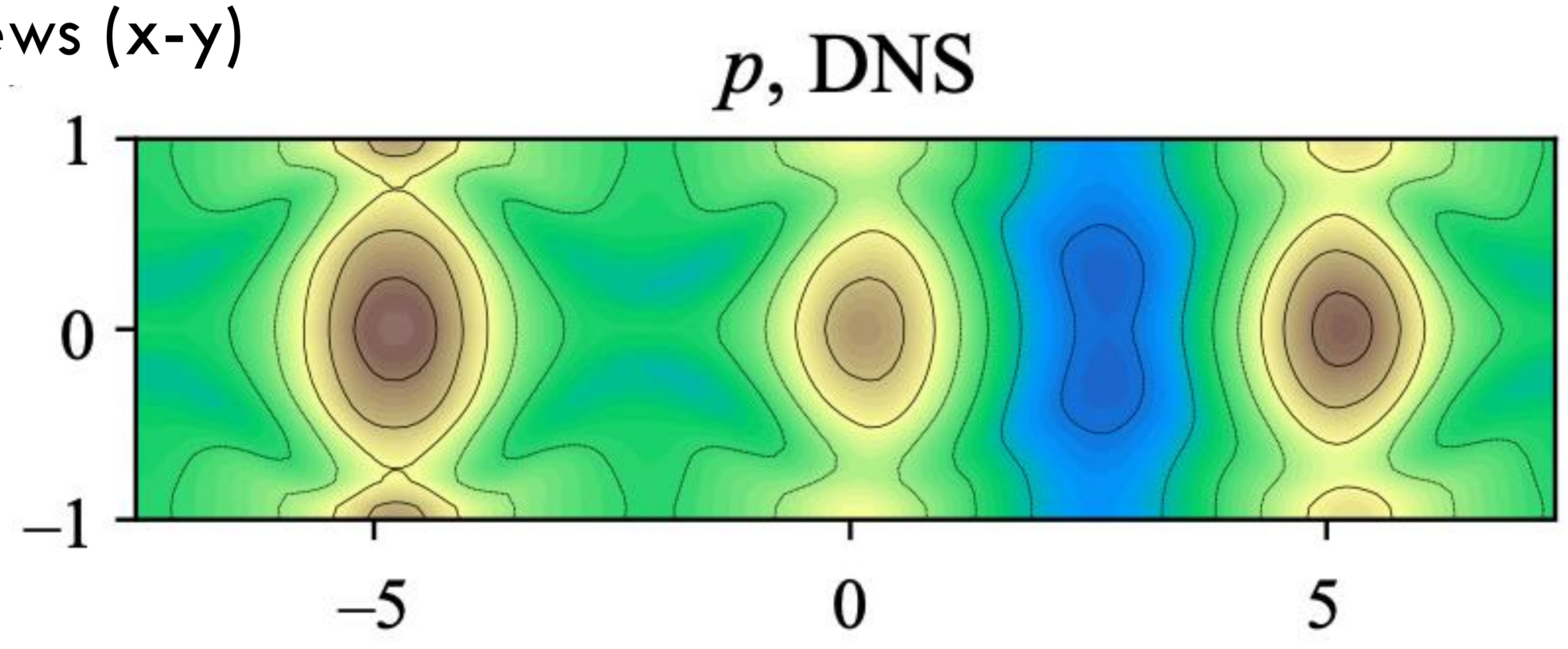


Validation of the PINN with DNS

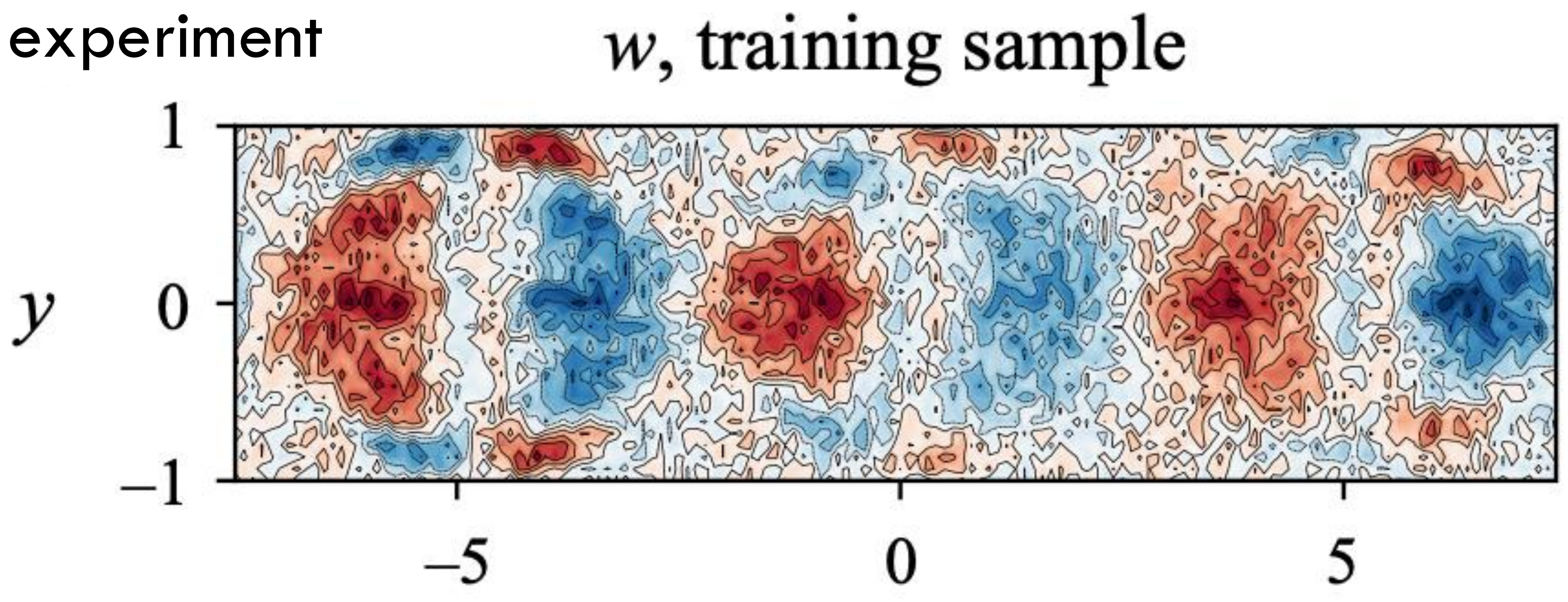
Top views (x-y)

Ground truth





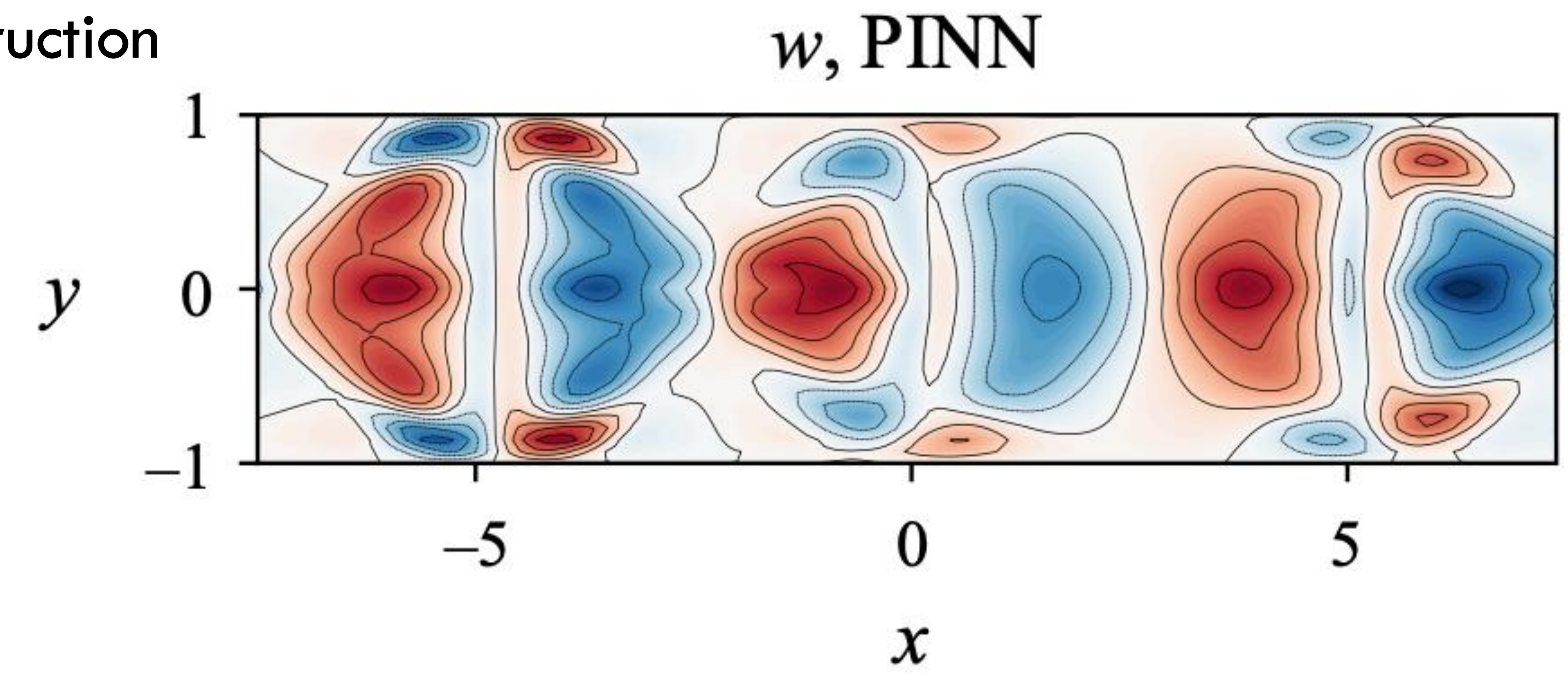
Synthetic experiment

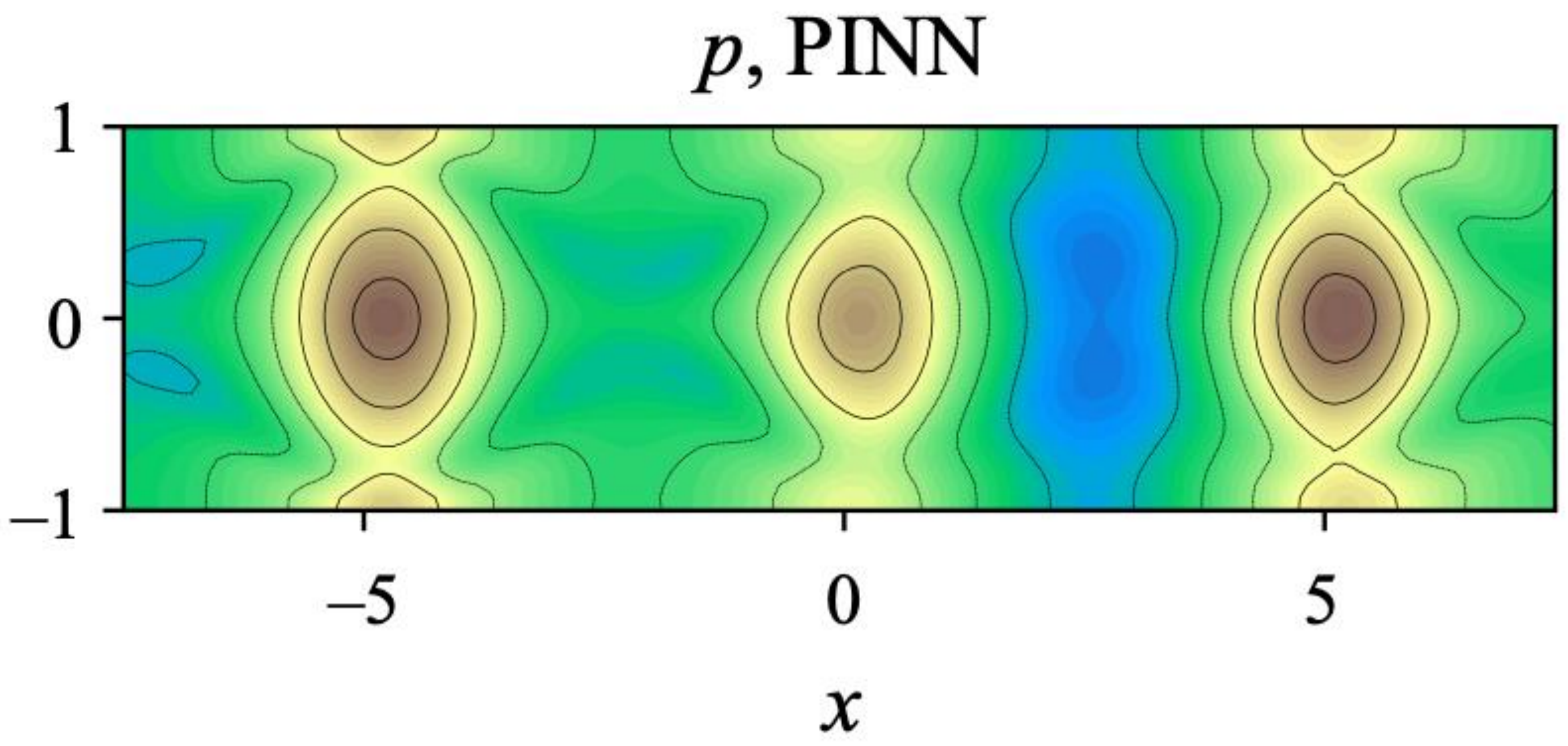


Training data to mimic experiments:

- Down-sample resolution
- Mimic scanning (not shown here)
- Add 5% noise
- Exclude pressure

Reconstruction





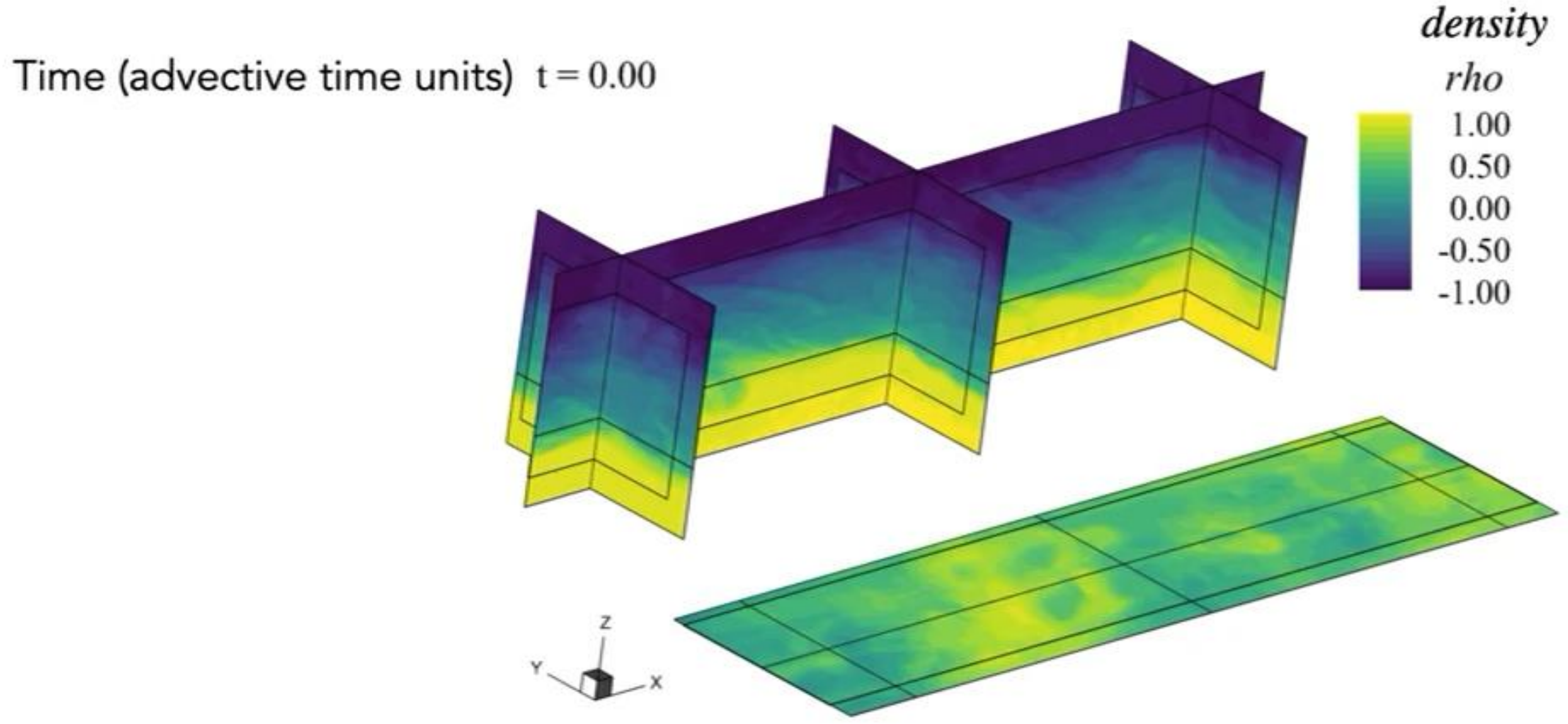
PINN for stratified turbulence?

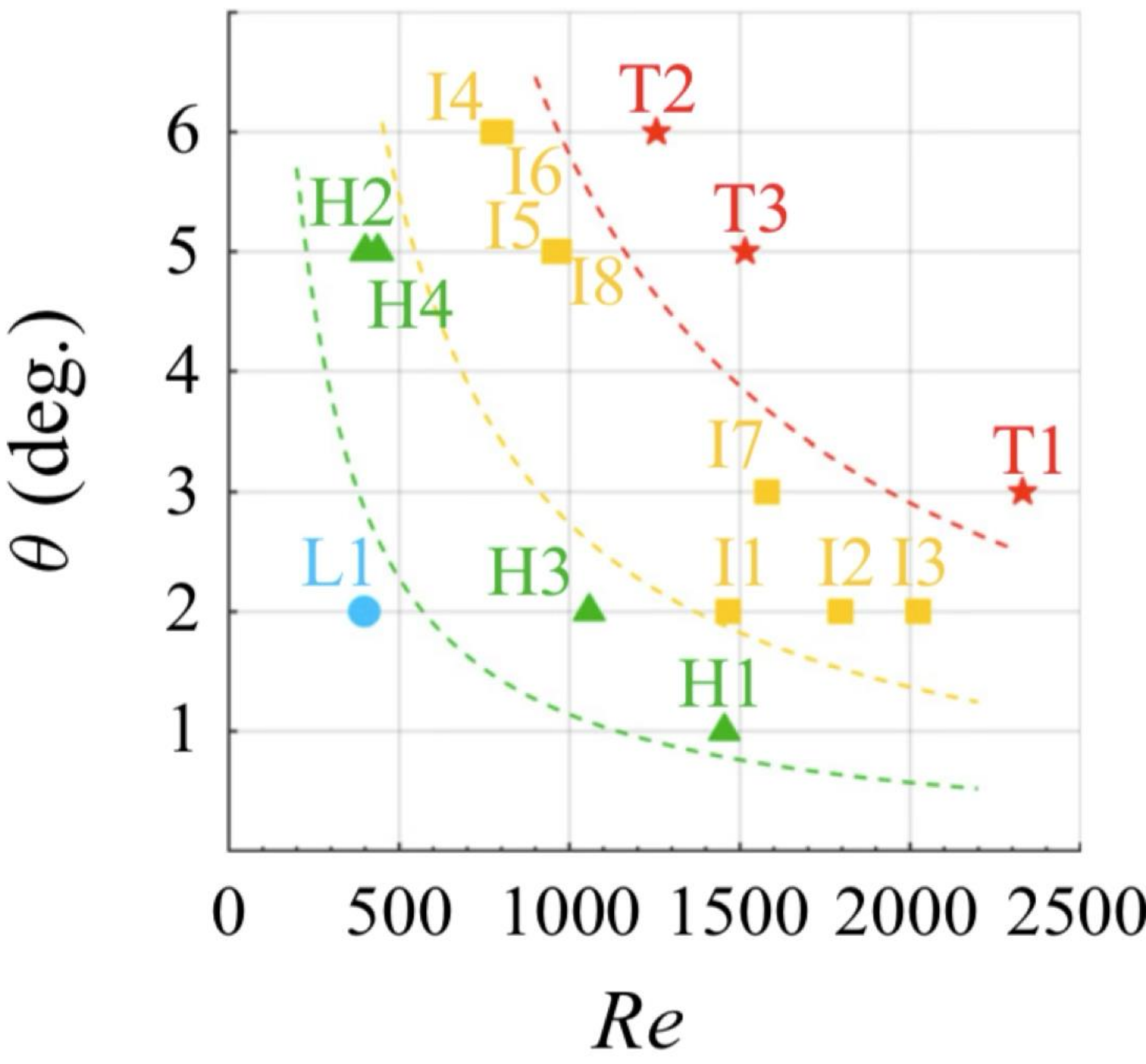
Still out of reach…
Expensive, risk of overfitting, small-scales below experimental resolution?

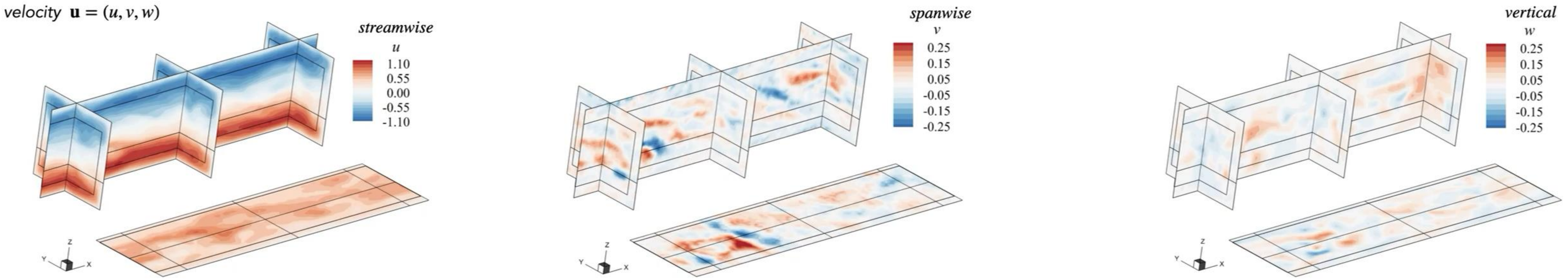
Dataset

T2

$\theta = 6^{\circ}$
$Re^s = 1030$
$Ri_b^s = 0.134$







Source: Lefauve, A. & Linden, P.F. 2022 Research data supporting "Experimental properties of continuously-forced, shear-driven, stratified turbulence" [Dataset]. doi.org/10.17863/CAM.75370.

Conclusions

The Stratified Inclined Duct mimics an ‘infinite’ gravity current in the laboratory, with Holmboe waves, intermittency, turbulence

Physics Informed Neural Networks augment experimental data (PIV+LIF), ensuring the reconstruction obeys physical laws

Result 1: Undoing the spanwise distortion

The PINN is naturally suited to take in 3D scanned data and fix it

Result 2: Reduced noise and improved spatio-temporal resolution

Better coherent structures, entrainment, and energy budgets

Result 3: Access to the latent pressure field

Useful because favourable vs adverse pressure gradient affects the transition to turbulence

BUT so far unsuccessful on turbulent flows!

J. Fluid Mech. (2024), *vol.* 981, R1, doi:10.1017/jfm.2024.49

JFM RAPIDS

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